ACKNOWLEDGMENTS

The author would like to take this opportunity to thank Dr D. Mukhedkar of Ecole Polytechnique of Montréal for his continued guidance, interest and support; Dr J.F. Lindsay of Concordia University of Montréal for his goodwill in reading and commenting on the text and Dr V. Ramachandran, also of Concordia University of Montréal for his constructive suggestions.

Finally the author wishes to express his appreciation to his wife, Chryssanthi, for her skillful typing of the manuscript and her patience and understanding during authorship.

PREFACE

The first detailed reliability studies were undertaken as early as 1938[1] when S.M. Dean in his paper "Consideration Involved in Making System Investment for Improved Service Reliability", introduced reliability as a factor to consider when designing a power system.

In the field of electronics, the reliability concept was intorduced after 1945 and it was used by the armed forces to improve the performance of communications and navigational systems.[2] In the early 60's the reliability was introduced to almost all engineering disciplines.

This report will examine the different methods that have been developed to predict the reliability of engineering systems. The number of methods presented in this report by no means implies that there are not other methods available. It can be stated however, that the methods represent the most popular ones among design engineers.

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CHAPTER I

INTRODUCTION

1.1 Introduction

The primary objective of this report is to collect and illustrate techniques that have been applied to predict the reliability of engineering systems. This includes reliability predictions for electronic, power, mechanical and civil engineering systems.

This first chapter of the report is devoted to reliability definition.[3] Some of the terms defined in this chapter are the mean time to failure (MTTF), the mean time between failures (MTBF) and the failure rate. The reliability of electronic systems is the subject of the second chapter. This particular area of reliability has received increasing attention during the past twenty years as electronic systems have grown complicated and their operation has become, in some case, imperative.[4] (Radar applications, nuclear reactor controls). In this report one method of predicting the reliability of electronic system is illustrated. It is called the part failure rate method and it is been proved very accurate.

The reliability of power systems is covered in chapters three through five.

Chapter three deals with the reliability of power distribution systems. Chapter four with the reliability of power transmission systems and finally chapter five with the reliability of power system generation.

The methods employed here are the frequency and duration method for distribution systems, the Markov chains method and the approximate method for transmission systems and the frequency and duration by recursive techniques method for power generation systems.

The last chapter of this report is concerned with the reliability of mechanical and civil engineering systems. Two distributions are usually employed to calculate the reliability of these systems, the Weibull and the normal distributions.[5] Both distributions are equally important and reliable in their results. Finally examples and tables are given throughout the chapter to illustrate the various techniques. Some of the examples represent practical applications and all tables are extracted from accepted standard tables.

1.2 Reliability

The following sections of this chapter define the different reliability indexes and their mathematical relationships.[6]

The National Aeronautics and Space Administration (NASA) defines reliability as the probability of a device performing adequately for the period of time intended under the operating conditions encountered. Thus, the reliability of a computer might be given as 90 per cent over a 1,000-hour period, with an ambient temperature of 20°C and no vibration. Here, the ambient temperature of 20°C and the no-vibration environment are the encountered conditions.

It is obvious from the definition, that the reliability is always a probability associated with a no-failure performance of a device. This is why some scientists define the reliability as the probability of survival and others as the probability that a given system will perform as anticipated.

Reliability cannot be used to predict discrete events, that is, it cannot predict when a failure will take place. Instead its main function is to predict the average number of failures anticipated over a period of time.

Reliability can be classified as predicted, inherent and demonstrated. Predicted reliability is the reliability calculated, based on the designs before assembly or test of the final system. Inherent reliability results from intrinsic values of functions and designs and can change only with changes in design or degradation during operation. Demonstrated reliability is the reliability determined from test and/or field performance.

Since reliability is defined as a probability, its calculation is a form of applied mathematics. This report, however, will not examine the probability mathematics of reliability, instead, it will study the various techniques in use to obtain the reliability of engineering systems, emphasizing the reliability of the electrical and electronic systems. (Chapters II to V inclusively).

It can be shown then, that the general reliability function is given by:

$$R(t) = \exp\left\{ - \int_0^t \lambda(t)\, dt \right\} \tag{1.1}$$

where,

$R(t)$ = the reliability

and,

$\lambda(t)$ = time dependant failure rate.

If $\lambda(t) = \lambda$ = constant, then eq.(1.1) becomes:

$$R(t) = e^{-\lambda t} \tag{1.2}$$

It will be shown later that letting $\lambda(t) = \lambda$ = constant, is a very reasonable and practical approximation.

1.3 The Mean Time to Failure (MTTF)

For nonmaintained systems MTTF is a measure of the expected time the system is in an operable state before all the equipments reach a failed state. The term nonmaintained implies, that the failed equipments are not repaired but they are replaced with new and good equipments.

The value of MTTF can be calculated as follows. Let a set of n items be tested until all have failed, the times to failure being $t_1, t_2, t_3, \ldots, t_n$. Then the observed MTTF is given by:

$$M_T = \frac{1}{n} \sum_{i=1}^{n} t_i \tag{1.3}$$

The MTTF can also be expressed in terms of the failure rate as:

$$M_T = \frac{1}{\lambda} \tag{1.4}$$

where λ = failure rate = constant. Equations (1.3) and (1.4), then, combined give,

$$M_T = \sum_{n=1}^{n} \frac{1}{n\lambda} \quad (1.5)$$

where, n= number of equipments.

1.4 Mean Time Between Failures (MTBF)

Mean time between failures is defined as the average time between two successive component failures. These are not necessarily failures of identical components and generally will not be. The MTBF of a system may be measured by testing the system for a total period of time T during which n faults occur. Each fault is repaired and the equipment is put back on test, the repair time being excluded from the total time T. The observed MTBF is then given by:

$$M_B = \frac{T}{n} \quad (1.6)$$

where: M_B = MTBF

T = total time

n = number of failures

The MTBF can also be expressed as the reciprocal of the failure rate, provided that the failure rate is constant, that is,

$$M_B = \frac{1}{\lambda} \tag{1.7}$$

where: λ = failure rate = constant.

Equations (1.7) and (1.4) imply that MTTF = MTBF which is the case when λ is constant. For most engineering applications, this is a very practical assumption because it simplifies calculations but unfortunately, it contributes to confusion between the concept of MTTF and MTBF.

1.5 Failure Rate

The failure rate of a system is defined as the number of failures observed per unit measure of life (i.e. cycle, time).[7] It is usually denoted by the Greek letter λ and it is measured in failures per 10^6 hours for electronic systems and in failures per year in power systems. The failure rate λ is related with reliability according to equations (1.1) and (1.2). Eq.(1.1) is the general form and eq.(1.2) is a special case of eq.(1.1) and it applies when the failure rate is constant. In actual life however, λ is a function of time as shown in fig.(1.1).

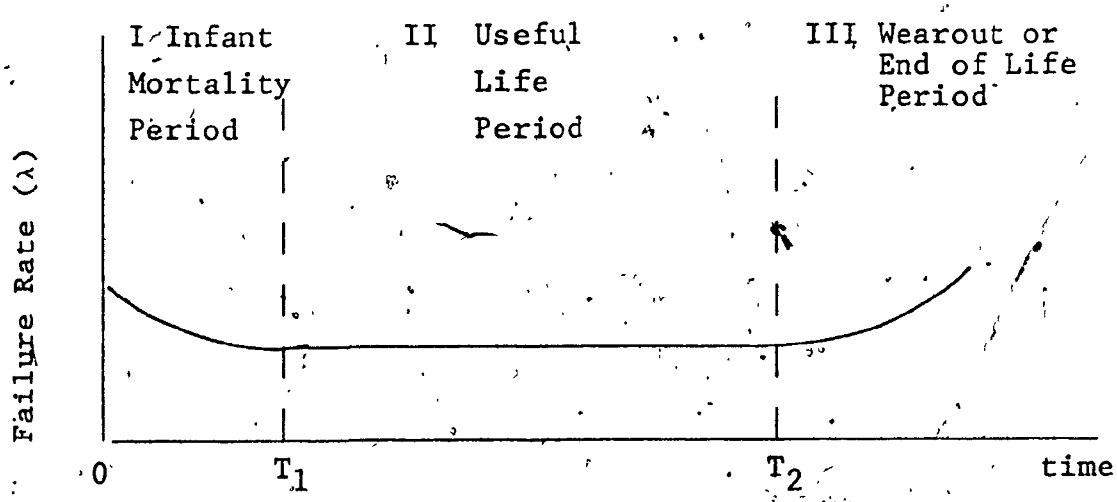

Fig.(1.1) Failure rate "bath tub" curve.

In fig.(1.1), the time interval 0 to T_1, is known as the infant mortality period. During this time period, the failure rate is quite high and this is due to built-in-flaws, faulty workmanship, transportation damage and installation errors. Many manufacturers provide a "burn-in" period for their product, prior to delivery, which helps to eliminate a high portion of the initial failures and assists in establishing a high level of operational reliability. The second time interval, that is, the interval between T_1 and T_2 in fig.(1.1), is called the useful time period and it is characterized mainly by the occurence of stress related failures. The failure rate at this stage reaches its lowest level and it is relatively constant. Since the useful life period covers most of the life span of the system, it is reasonable to assume that letting λ to be constant at all times is a fairly good approximation. The third and final life period is called wearout period. It occurs when the system reaches the point where the failure rate starts to increase noticeably. When the failure rate due to wearout becomes unacceptably high, replacement or repair of the system should be made.

The importance and the numerous applications of the failure rate in reliability estimates, will be apparent in chapter II where the reliability of electronic systems is studied. It will be seen in this chapter, that the knowledge of failure rate values is sufficient to obtain very accurate reliability results.

1.6 Systems Reliability

The previous definitions are completely general and may apply to elements as well as to systems, provided that the system is considered to be an one-element quantity. This section studies the reliability of systems that are composed by elements whose reliabilities are somehow known.[8,9] These elements may be connected in series and/or in parallel, a combination of both or none of the above.

1.6.1 Series Systems Reliability

Fig. (1.2) shows n elements connected in series.

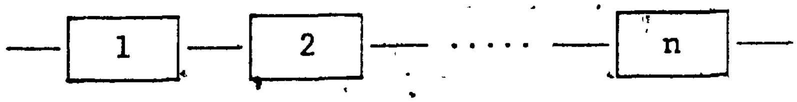

Fig. (1.2). Series elements

Each element has a reliability $R_i(t)$ $i = 1, 2, \ldots, n$. All elements must be properly working in order for the system to be operating. If one element fails, the system will fail also. The overall system reliability, then is:

$$R(t) = R_1(t)\, R_2(t) \,\ldots\ldots\, R_n(t) \tag{1.8}$$

where $R(t)$ is the total reliability and $R_i(t)$ is the reliability of the ith element. If the failure rate for each of the elements was constant, equ. (1.8) would be:

$$R(t) = e^{-\lambda t} = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \,\ldots\ldots\, e^{-\lambda_i t} \ldots\ldots e^{-\lambda_n t}$$

$$= e^{-(\lambda_1 + \lambda_2 + \cdots + \lambda_i + \cdots + \lambda_n)t}$$

$$= \exp\left\{ -t \sum_{i=1}^{n} \lambda_i \right\} \tag{1.9}$$

1.6.2 Reliability of Parallel Systems

Before attempting to calculate the reliability of parallel systems, the concept of unreliability shall be introduced. Unreliability is the probability that the system will not meet the specified performance requirements under a given set of conditions. It is related to reliability as follows,

$Q + R = 1$ where, Q = unreliability and

R = reliability.

Fig.(1.3) shows n elements connected in parallel.

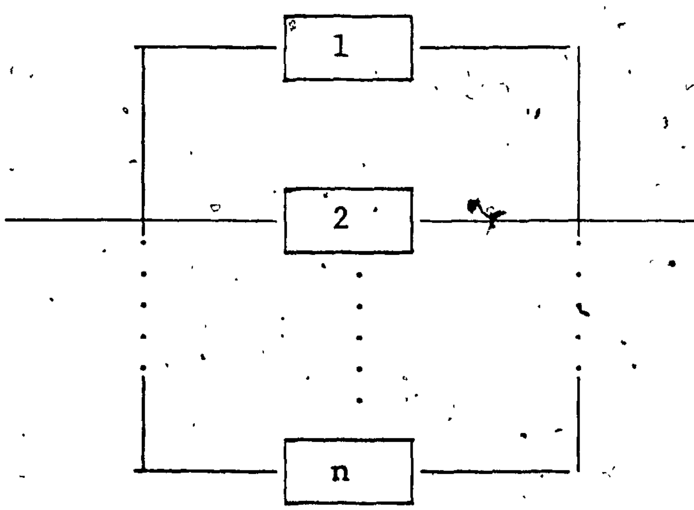

Fig.(1.3). n parallel systems.

All equipments operate simultaneously and they are independent. The system fails when all equipments fail. Thus, the unreliability of the system is equal to the product of the individual unreliabilities, i.e.:

$$Q_i(t) = Q_1 Q_2 \ldots Q_i \ldots Q_n = \prod_{i=1}^{n} Q_i(t) \qquad (1.10)$$

where, $Q_i(t)$ = the unreliability of the system and

Q_i = the unreliability of the ith element.

Thus, the reliability is,

$$R_i(t) = 1 - Q_i(t) = 1 - \prod_{i=1}^{n} Q_i(t) \qquad (1.11)$$

or,

$$R(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t)) \qquad (1.12)$$

Finally for n equal components, eq.(1.12) gives:

$$R(t) = 1 - Q_i = 1 - Q^n = 1 - (1 - e^{-\lambda t})^n \qquad (1.13)$$

1.7 Redundancy

The reliability of a system can be significantly improved through the use of redundancy.[10, 11] Redundancy involves one or more alternate paths into the system through addition of parallel elements. Depending on the specific application, a number of approaches are available to improve reliability using redundant design. These design approaches can be classified on the basis of how the redundant elements are introduced into the circuit to provide a parallel path. In general, there are two major classes of redundancy, active redundancy and standby redundancy.

Active redundancy requires no external components to detect a failure in the system. Once an element of the system fails, the redundant apparatus is switched on automatically. Standby redundancy requires external components to detect a failure, make a decision and switch the apparatus on, to replace the failed path.

Techniques related to active redundancy include the simple parallel redundancy, the bimodal parallel/series redundancy and the bimodal series/parallel redundancy. Each of the above will be examined and analyzed.

1.7.1 Simple Parallel

Simple parallel redundancy is the simplest form of redundancy. It consists of a simple parallel combination of n elements. If any element fails, identical paths exist through parallel redundant elements. See fig. (1.4). The reliability of this simple parallel configuration is:

$$R = 1 - (1 - e^{-\lambda t})^{n} \tag{1.14}$$

where, n = number of parallel elements,

and, λ = failure rate.

Note, that eq.(1.14) is the same as eq.(1.13) as expected.

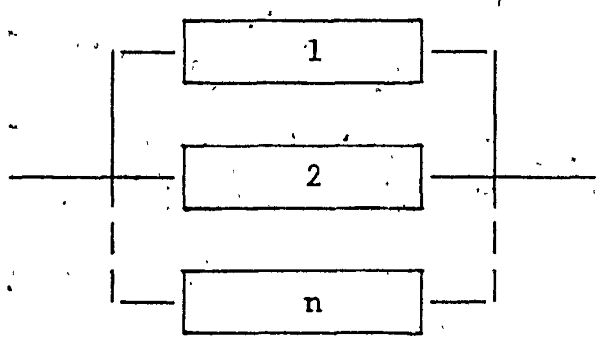

Fig.(1.4) Simple parallel redundancy.

Fig.(1.5) shows the gain in reliability as n increases.

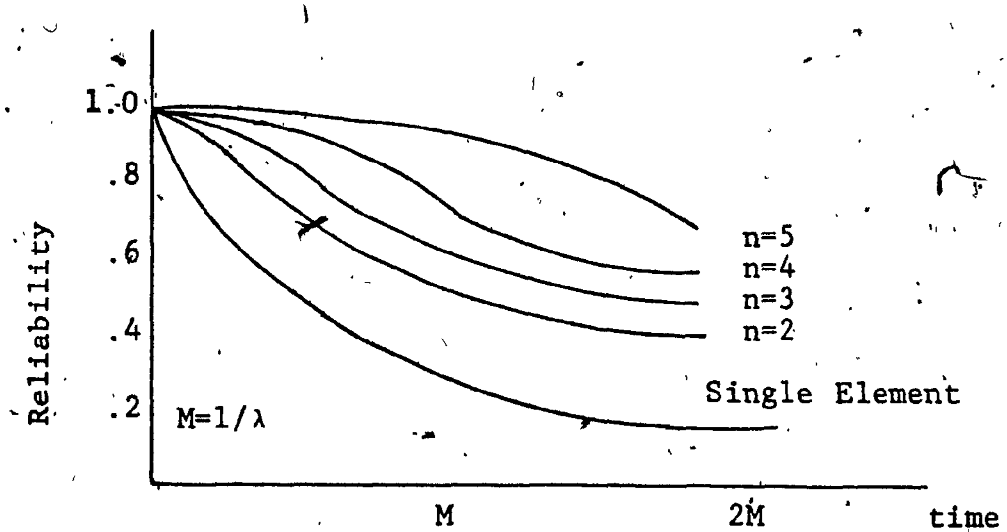

Fig.(1.5) Reliability function for simple parallel redundancy

Some of the advantages of this technique, include simplicity, gain in reliability and applicability to both analog and digital circuitry. The main disadvantages are, load sharing considerations and sensitivity to voltage variations across the elements.

1.7.2 Bimodal Parallel/Series Redundancy

This series connection of parallel redundant elements, provides protection against open circuit failure modes. It also provides significant gain in reliability at the part or stage level for short missions times. Its main disdvantage is that it is difficult to design. Fig.(1.6) shows a bimodal parallel/series configuration.

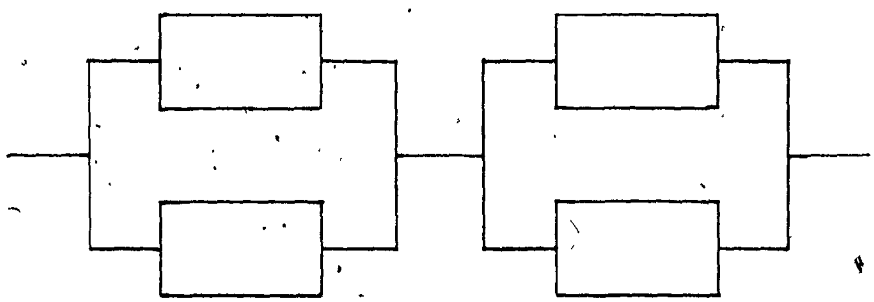

Fig.(1.6) Bimodal parallel/series redundancy

Assuming that all elements are identical, the reliability is,

$$R = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t} \qquad (1.15)$$

1.7.3 Bimodal Series/Parallel Redundancy[5]

Fig.(1.7) shows a parallel connection of series elements. This redundant configuration increases the reliability of the system because it protects it against the short circuit failure modes.

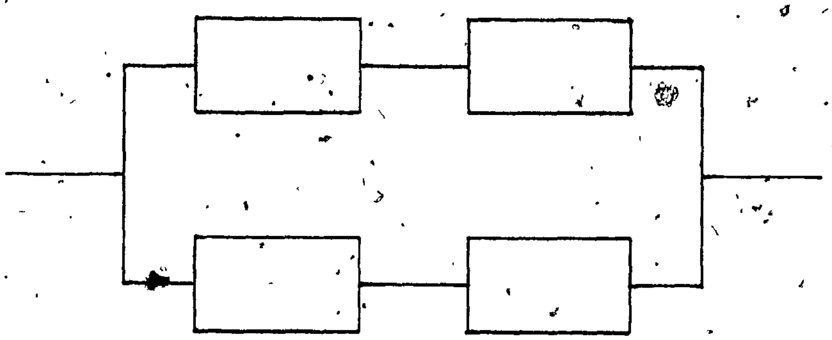

Fig.(1.7) Bimodal series/parallel redundancy

Assuming that all elements are similar, its reliability expression has as follows:

$$R = 2e^{-\lambda t} - e^{-2\lambda t} \tag{1.16}$$

A comparison of the reliability of the two bimodal configurations and the non-redundant single element system is shown in fig.(1.8). It is evident that the reliability gain when employing bimodal redundancy, is greater during the early life of the element. This is why bimodal redundancy is recommended for short time missions.

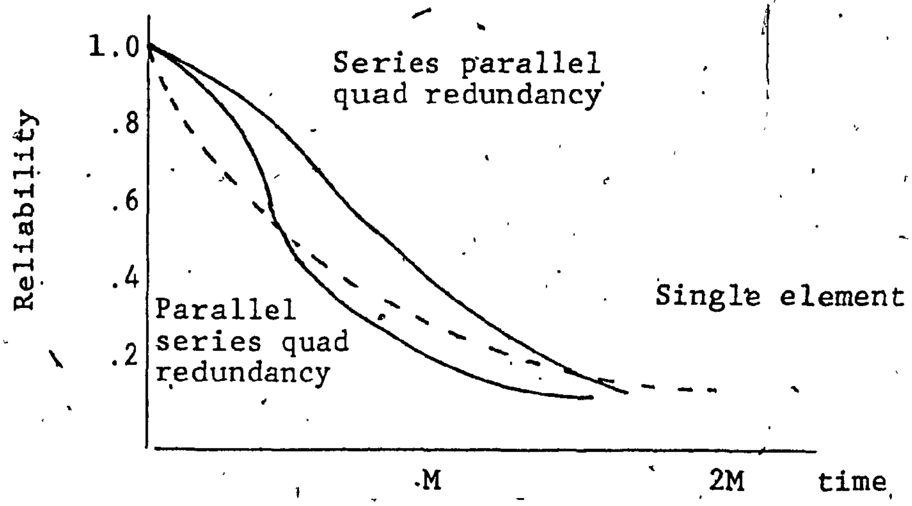

Fig. (1.8) Reliability function for bimodal configurations

1.7.4 Standby Redundancy

The standby redundancy technique requires that a particular redundant element of a parallel configuration can be switched into an active circuit by connecting outputs of each element to switch poles. Two switching configurations are possible.

a) The element may be isolated by the switch until switching is completed and power applied to the element in the switching operation.

b) All redundant elements are continuously connected to the circuit and a single redundant element activated by switching power to it.

Both configurations are shown in fig.(1.9).

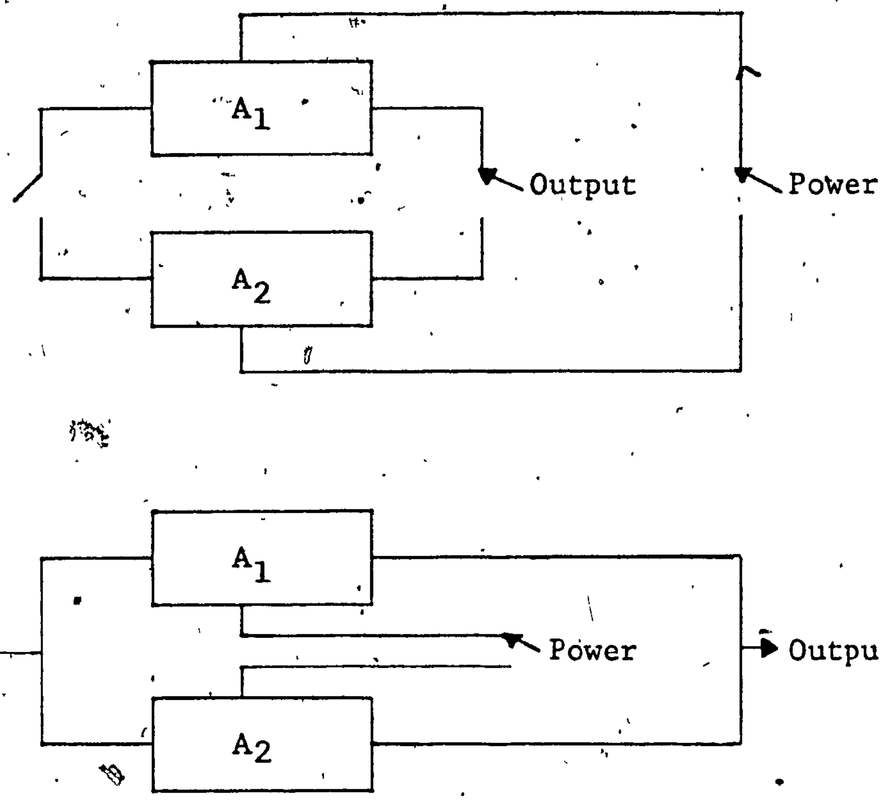

Fig. (1.9) Standby redundancy.

The reliability function for both cases is:

$$R = e^{-\lambda t} \left(1 + \frac{\lambda}{\lambda_s} \left(1 - e^{-\lambda_s t} \right) \right) \tag{1.17}$$

where, λ_s = failure rate of switching function.

CHAPTER II

THE RELIABILITY OF ELECTRONIC SYSTEMS

2.1 Introduction

One of the most effective and realistic methods developed to establish reliability figures for electronic equipment is the part failure rate method. The method has been applied extensively by the military engineers in their effort to design highly reliable equipments and systems. It emphasizes the calculation of the failure rate of the parts that comprise a system which in turn can be translated in reliability using the appropriate formulas.

To obtain the part failure rate, the base failure rate is calculated first and the result is then modified by correction factors. These correction factors or π factors account for such effects as the environment, the application, the complexity, the quality level, the temperature and the voltage stress. The definition of the factor and their specific application will be given in the next section of this chapter.

In this chapter, methods to find the failure rates for diodes, transistors, resistors, capacitors, lasers, and microelectronic devices will be presented.[12] Illustrative exmples will also be given to appreciate the usefulness and the applicability of the method.

2.2 The π factors

The most important π factors are the environment factor π_E and the quality factor π_Q. The π_E factor accounts for the influence of environmental stresses on the electronic parts. Its value varies with the environment in which the part operates, as shown in tables (2.1) and (2.2). The first table describes π_E whereas the second lists its values against different environments for various electronic parts. The quality factor π_Q accounts for effects of different quality levels. These quality levels are predetermined by the manufacturer. Different letters are used to describe each level. Table (2.3) lists the indicating levels as well as the value of π_Q for the different categories of electronic equipment. The resistance factor π_R adjusts the equation of failure rate for resistors for the effect of the ohmic values.

Table (2.4) indicates the values of π_R as the resistance changes.

Environment	π_E Symbol	Nominal Environmental Conditions
Ground, Benign	GB	Nearly zero environmental stress with engineering operation and maintenance.
Space, Flight	SF	Earth orbital. Approaches Ground, Benign conditions without access for maintenance.
Ground, Fixed	GF	Conditions less than ideal to include installation in permanent racks with adequate cooling air, maintenance by military personnel and possible installation in unheated buildings.
Ground, Mobile (and Portable)	GM	Conditions more severe than those for GF; mostly for vibration and shock. Cooling air supply may also be more limited, and maintenance less uniform.
Naval, Sheltered	NS	Surface ship conditions similar to GF but subject to occasional high shock and vibration.
Naval, Unsheltered	NU	Nominal surface shipborne conditions but with repetitive high levels of shock and vibration.
Airborne, Inhabited	AI	Typical cockpit conditions without environmental extremes of pressure, temperature, shock and vibration.
Airborne, Uninhabited	AU	Bomb-bay, tail, or wing installations where extreme pressure, temperature and vibration cycling may be aggravated by contamination from oil, hydraulic fluid, and engine exhaust.
Missile, Launch	ML	Severe conditions of noise, vibration, and other environments related to missile, launch and space vehicle boost into orbit, vehicle re-entry and landing by parachute.

Table (2.1). Environmental symbol identification and description.

	π_E				
ENVIRONMENT	DISCRETE SEMICONDUCTORS	RESISTORS	CAPACITORS	LASERS	MICROELECTRONIC DEVICES
GB	1.0	1.0	1.0	0.2	0.2
SF	1.0	1.0	1.0	0.2	0.2
GF	5.0	2.0	2.0	1.0	1.0
AI	25.0	4.0	4.0	5.0	4.0
NS	25.0	5.0	4.0	5.0	4.0
GM	25.0	7.0	4.0	5.0	4.0
NU	25.0	7.5	8.0	5.0	6.0
AU	40.0	8.0	10.0	8.0	5.0
ML	40.0	15.0	15.0	8.0	10.0

Table (2.2). π_E values for different electronic components.

DISCRETE SEMICONDUCTORS		RESISTORS		CAPACITORS		MICROELECTRONIC DEVICES	
LEVEL	π_Q	LEVEL	π_Q	LEVEL	π_Q	LEVEL	π_Q
JANTXU	.2	M	1.0	L	1.5	A	1
JANTX	.4	P	.3	M	1.0	B	2
JAN	2.0	R	.1	P	.3	B-1	5
LOWER	10.0	S	.03	R	.1	B-2	10
PLASTIC	20			S	.03	C	16
						C-1	90
						D	150
						D1	300

Table (2.3). Quality factor π_Q.

Resistance Range kΩ	π_R
Up to 100 kΩ	1.0
> 0.1 Megohm to 1 Megohm	1.1
> 1.0 Megohm to 10 Megohm	1.6
> 10 Megohm	2.5

Table (2.4). Resistance factor π_R.

For discrete semiconductors (diodes, transistors) applications, the following factors are required:

a) π_A - Application factor. This factor accounts for effects of application in terms of circuit functions. Table (2.5) shows how π_A changes for different applications.

Application	π_A	
	Diodes	Transistors
Small Signal (< 500 mA)	1.0	-
Logic Switching	0.6	0.7
Power Rectifier (>500 mA)	1.5	-
Linear	-	1.5
High Frequency (freq.>400 MHz and average power <300 mW)	-	5.0

Table (2.5). π_A application factor.

b) π_P - Power rating factor. This factor accounts for effect of maximum power or current ratings. Its values as a function of power ratings are shown on the following table (2.6).

Transistors		Diodes	
Power Rating (watts)	π_P	Current Rating (A)	π_P
< 1	1	< 1	1
> 1 to 5	1.5	> 1 to 3	1.5
> 5 to 20	2.0	> 3 to 10	2.0
> 20 to 50	2.5	> 10 to 20	4.0
> 50 to 100	5.0	> 20 to 50	10.0

Table (2.6). π_P power rating and current rating factor.

c) π_C - Complexity factor. This factor accounts for effect of multiple devices in a single package. For transistors the π_C factor takes the following values:

Complexity	π_C
Single Transistor	1.0
Dual (Unmatched)	0.7
Dual (Matched)	1.2
Darlington	0.8
Dual Emitter	1.1
Multiple Emitter	1.2
Complementary Pair	0.7

Table (2.7). π_C complexity factor for transistor applications only.

d. π_{S2} - Voltage stress factor. It adjusts model for a second electrical stress (application voltage) in addition to power included within λ_b. Its value is given as:

$$\pi_{S2} = \begin{cases} 1.4 \times 10^{(.0133)S_2} & \text{for } S_2>22 \\ 0.3 & \text{for } S_2<22 \end{cases} \quad (2.1)$$

where, $S_2 = \dfrac{\text{Applied } (V_{CE})}{\text{Rated } (V_{CE})} \times 100$

V_{CE} = Collector-Emitter voltage.

Table (2.8) tabulates π_{S_2} for different values of S_2.

S_2 (percent)	π_{S_2}
100	3.0
90	2.25
80	1.65
70	1.2.
60	0.88
50	0.64
40	0.48
30	0.36
20	0.30
0	0.30

Table (2.8). π_{S_2} voltage stress factor for transistor applications only.

e) π_T - Temperature. This factor accounts for effects of temperature.

Finally for the computation of the failure rate of lasers the following π factors are necessary:

a) π_{CL} is the cleanliness factor. This factor accounts for the cleanliness of the couplings. Its value is 1 for rigorous cleanliness procedures, 30 when minimal precautions during opening are taken, and 60 for poor maintenance.

b) π_{COOL} is the flashlamp cooling factor. This factor depends on the cooling media and it has as follows for the different cooling media.

Cooling Media	π_{COOL}
Gas, Air	1.0
Gas, Inert	1.0
Liquid, Deionized Water	0.1
Liquid, Water-Glycol	0.1
Liquid, Fluorocarbon	0.1

Table (2.9). Flashlamp cooling factors, π_{COOL}

c) π_{REP} is the repetition rate factor and it converts pulse rate to time rate base for pulsed lasers. Its value as a function of pulse rate are shown in table (2.10).

Pulse rate	π_{REP}
1	3,600
5	18,000
10	36,000
15	54,000
20	72,000

Table (2.10). Pulse rate factors, π_{REP}

d) π_{OS} is the optical surface factor which is the sum of all active surfaces in the laser. Mirrors have usually two active surfaces.

2.3 Resistors Reliability

The general expression for the failure rate of resistors is:

$$\lambda_p = \lambda_b \left(\pi_E \times \pi_R \times \pi_P \right) \quad \text{(failures per } 10^6 \text{ hours)} \tag{2.2}$$

where,

λ_p = the resistor part failure rate,

λ_b = base failure rate, and

π_E, π_R, π_P as defined in section (5.2) along with their values.

The base failure is given as:

$$\lambda_b = A \cdot \exp\left(B\left(\frac{T + 273}{N_T} \right)^G \right) \exp\left(\left(\left(\frac{N}{N_S} \right) \left(\frac{I + 273}{273} \right)^J \right)^H \right) \tag{2.3}$$

where,

T = ambient temperature in degrees Centigrade,

N_T = temperature constant,

B = shaping parameter,

G, H, J = accelaration constant,

N_S = stress constant

S = the electrical stress and it is defined as the ratio of operating power to rated power.

Table (2.11) shows the values of λ_b as a function of temperature and electrical stress. It is evident from this table that the electrical stress ratio increases the failure rate quite rapidly as it increases, whereas temperature increase does not increase the failure rate as much (within operating limits).

Having defined all parameters, the failure rate of the resistor can now be found and once the failure rate is established and assuming that it is constant, the reliability of the resistor can be calculated according to equation (1.2).

As an example, consider a 10 kΩ resistor being part of an electronic equipment installed in a cockpit of an airplane. Its rated power is 0.6W and the resistor is of quality level M. The resistor ambient temperature is 50° C and is dissipating 0.3 W. It is desired to find the reliability of the resistor over a ten year period. The failure rate must be calculated first and then the reliability. The failure rate is given by:

$$\lambda_p = \lambda_b \text{ X } \pi_E \text{ X } \pi_R \text{ X } \pi_Q$$

The stress ratio S is,

$$S = \frac{P_{APPLIED}}{P_{RATED}} = \frac{0.3}{0.6} = 0.5$$

and from table (2.11) for $S = 0.5$ and $T = 60^\circ C$, λ_b is found to be 0.0014. π_E for airborne inhabited environment is 4 and π_Q is 1 for level M. Finally π_R for 10 kΩ is 1.0. Therefore,

$$\lambda_p = 0.0014 \text{ X } 4 \text{ X } 1 \text{ X } 1 = 0.0056 \text{ failure per } 10^6 \text{ hours.}$$

Then the reliability over a 10 years life span is:

$$R = e^{-\lambda t} = \exp(\frac{0.0056}{10^6} \text{ X } 8760 \text{ X } 10) = 0.99943$$

T (°C)	RATIO OF OPERATING TO RATED POWER									
	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.00007	.00009	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003
5	.00009	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0004
10	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
20	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007
25	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0009
30	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0009	.0011
35	.0002	.0003	.0003	.0004	.0005	.0006	.0008	.0009	.0011	.0014
40	.0003	.0003	.0004	.0005	.0006	.0008	.0009	.0011	.0014	.0017
45	.0003	.0004	.0005	.0006	.0008	.0009	.0011	.0014	.0017	.0021
50	.0004	.0005	.0006	.0008	.0009	.0011	.0014	.0017	.0021	.0026
55	.0005	.0006	.0007	.0009	.0011	.0014	.0017	.0021	.0026	.0032
60	.0006	.0007	.0009	.0011	.0014	.0017	.0021	.0026	.0032	.0039
65	.0007	.0009	.0011	.0014	.0017	.0021	.0026	.0032	.0039	.0048
70	.0009	.0011	.0013	.0016	.0020	.0025	.0031	.0039	.0048	.0059
75	.0010	.0013	.0016	.0020	.0025	.0031	.0038	.0047	.0059	
80	.0012	.0015	.0019	.0024	.0030	.0037	.0046	.0058		
85	.0015	.0019	.0023	.0029	.0036	.0045	.0057			
90	.0018	.0022	.0028	.0035	.0044	.0055				
95	.0021	.0027	.0034	.0043	.0054	.0067				
100	.0026	.0032	.0041	.0052	.0065					
105	.0031	.0039	.0049	.0062						
110	.0037	.0047	.0059							
115	.0044	.0056								
120	.0053									
125	.0063									
130										

Table (2.11). The base failure of resistors as a function of temperature and stress ratio.

2.4 Failure Rate of Capacitors

The general model for the failure rate of capacitors is as follows:

$$\lambda_p = \lambda_b \times \pi_E \times \pi_Q \qquad (2.4)$$

where, π_E and π_Q are defined and given in section 2.2 and λ_b is the base failure rate given as:

$$\lambda_b = A \left[\left(\frac{S}{N_S}\right)^H + 1 \right]^B \exp \left(\left(\frac{T + 273}{N_T} \right)^G \right) \qquad (2.5)$$

where,

A is an adjustment factor,

S is the ratio of operating voltage,

N_S is a stress constant,

exp is the natural logarithmic base, 2.178,

T is the operating ambient temperature in degrees Celsius,

N_T is a temperature constant,

B is a shaping parameter,

G and H are acceleration constants.

Tables have been constructed to calculated λ_b as a function of ambient temperature and the ratio S of operating to rated voltage.

Table (2.12) shows the different values of λ_b in terms of T and S. Once λ_b is knwon, λ_p can be calculated according to eq.(2.4).

T	S, RATIO OF OPERATING TO RATED VOLTAGE									
(C)	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.00006	.00006	.00007	.0001	.0002	.0004	.0010	.0019	.0034	.0057
5	.00006	.00006	.00007	.0001	.0002	.0005	.0010	.0019	.0034	.0058
10	.00006	.00006	.00008	.0001	.0002	.0005	.0010	.0020	.0035	.0060
15	.00006	.00007	.00008	.0001	.0002	.0005	.0011	.0020	.0037	.0062
20	.00007	.00007	.00008	.0001	.0002	.0005	.0011	.0021	.0039	.0065
25	.00007	.00007	.00009	.0001	.0002	.0006	.0012	.0023	.0041	.0070
30	.00008	.00008	.0001	.0001	.0003	.0006	.0013	.0025	.0045	.0076
35	.00009	.00009	.0001	.0001	.0003	.0007	.0015	.0029	.0051	.0086
40	.0001	.0001	.0001	.0002	.0004	.0008	.0017	.0033	.0060	.010
45	.0001	.0001	.0001	.0002	.0005	.0010	.0022	.0041	.0074	.012
50	.0001	.0001	.0002	.0003	.0006	.0014	.0028	.0054	.0097	.016
55	.0002	.0002	.0002	.0004	.0009	.0020	.0041	.0077	.0013	.023
60	.0003	.0003	.0004	.0007	.0015	.0031	.0064	.012	.021	.036
65	.0006	.0006	.0008	.0013	.0027	.0057	.011	.022	.039	.066

Table (2.12). Base failure for capacitors.

Finally the reliability can be obtained applying eq.(1.2). As an exmple consider a capacitor rated at 400 V DC being used in a space flight. The ambient temperature is 30°C and the applied voltage is 28 V DC with 120 rms, 60 Hz. The quality level of the capacitor is L. It is required to find the reliability of the capacitor over a

70,000 hours interval. First the stress ratio S must be determined, hence,

$$S = \frac{28 + 120\sqrt{2}}{400} = 0.4$$

Thus, for $T = 30^{o}C$ and $S = 0.4$, λ_b is found to be 0.0001 failures per 10^6 hours. Then, the failure rate for $\pi_E = 1$ and $\pi_Q = 1.5$ is:

$$\lambda_p = 0.0001 \text{ X } 1 \text{ X } 1.5 = 0.00015 \text{ failures per } 10^6 \text{ hours.}$$

and the reliability is,

$$R = \exp\left(- \frac{0.00015}{106} \text{ X } 70,000 \right) = 0.99990$$

2.5 Transistors and Diodes

The general failure rate model for transistors and diodes is:

$$\lambda_p = \lambda_b\ (\pi_E \text{ X } \pi_A \text{ X } \pi_Q \text{ X } \pi_{S_2} \text{ X } \pi_C) \text{ failures per } 10^6 \text{ hours} \qquad (2.6)$$

where the various factors are defined in section 2.2. The equation for the base failure, λ_b is:

$$\lambda_b = A \exp\left(\frac{N_T}{273 + T + (\Delta T)S}\right) \exp\left(\frac{273 + T + (\Delta T)S}{T_M}\right)^P \qquad (2.7)$$

where,

A is a failure rate scaling factor,

exp is the logarithm base, 2.718,

T_M, N_T, P are shaping parameters,

T is the operating temperature in degrees Celsius,

ΔT is the difference between maximum allowable temperature with no junction current or power and the maximum allowable temperature with full rated junction current or power,

S is the stress ratio of operating electrical stress to rated electrical stress.

The values for the constant parameters are shown in table(2.13). The resulting base failure for transistors and diodes as function of temperature and electrical stress are shown in tables (2.14) and (2.15) respectively. These tables are based on the typical maximum junction temperatures of 100°C for Germanium devices and 175°C for Silicon devices.

It must be added here that the failure rate of thyristors is slightly different than that of diodes, namely the failure rate of thyristors is given as:

$$\lambda_p = \lambda_b \times \pi_Q \times \pi_E \times \pi_R \tag{2.8}$$

where, all terms have been defined previously, and can be taken approximately the same as those for diodes except the values of λ_b. The λ_b values are given in table (2.16).

Group	Part Type	λ_b Constants				
		A	N_T	T_M	P	T
Transistors	Si, NPN	0.13	-1052	448	10.5	150
	Si, PNP	0.45	-1324	448	14.2	150
I	Ge, PNP	6.5	-2142	373	20.8	75
	Ge, NPN	21.0	-2221	373	19.0	75
II	FET	0.52	-1162	448	13.8	150
III	Unijunction	3.12	-1779	448	13.8	150
Diodes	Si, Gen. Purpose	0.9	-2138	448	17.7	150
IV	Ge, Gen. Purpose	126	-3568	373	22.5	75
V	Zener/Avalanche	0.04	- 800	448	14	150
VI	Thyristors	0.82	-2050	448	9.5	150
VII	Microwave					
	Ge, Detectors	0.33	- 477	343	15.6	45
	Si, Detectors	0.14	- 392	423	16.6	125
	Ge, Mixers	0.56	- 477	343	15.6	45
	Si, Mixers	0.19	- 394	423	15.6	125
VIII	Varactor, Step Recovery & Tunnel	0.93	-1162	448	13.8	150

Table (2.13). Base failure constant parameters for diodes and transistors.

Table (2.14). Base failure of transistors.

T (^oC)	S									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.0034	0.0041	0.0048	0.0067	0.0067	0.0079	0.0095	0.011	0.014	0.018
10	0.0038	0.0046	0.0054	0.0064	0.0075	0.0089	0.010	0.013	0.017	0.023
20	0.0043	0.0051	0.0060	0.0071	0.0084	0.010	0.012	0.015	0.020	0.029
25	0.0046	0.0054	0.0064	0.0075	0.0089	0.010	0.013	0.017	0.023	0.033
30	0.0040	0.0057	0.0067	0.0079	0.0095	0.011	0.014	0.018	0.025	
40	0.0054	0.0064	0.0075	0.0089	0.010	0.013	0.017	0.023	0.033	
50	0.0060	0.0071	0.0084	0.010	0.012	0.015	0.020	0.029		
55	0.0064	0.0075	0.0089	0.010	0.013	0.017	0.023	0.033		
60	0.0067	0.0079	0.0095	0.011	0.014	0.018	0.025			
65	0.0071	0.0084	0.010	0.012	0.015	0.020	0.029			
70	0.0075	0.0089	0.010	0.013	0.017	0.023	0.033			
75	0.0079	0.0095	0.011	0.014	0.018	0.025				
80	0.0084	0.010	0.012	0.015	0.020	0.029				
85	0.0089	0.010	0.013	0.017	0.023	0.033				
90	0.0095	0.011	0.014	0.018	0.025					
95	0.010	0.012	0.015	0.020	0.029					
100	0.010	0.013	0.017	0.023	0.033					
105	0.011	0.016	0.018	0.025						
110	0.012	0.013	0.020	0.029						
115	0.010	0.013	0.017	0.023	0.033					
120	0.014	0.010	0.025							
125	0.015	0.020	0.029							
130	0.017	0.023	0.033							
135	0.018	0.025								
140	0.020	0.029								
145	0.023	0.033								
150	0.025									
155	0.029									
160	0.033									

T (°C)	S .1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.0005	.0007	.0010	.0014	.0019	.0025	.0033	.0043	.0057	.0082
10	.0006	.0009	.0013	.0017	.0023	.0030	.0039	.0052	.0072	.011
20	.0008	.0012	.0016	.0021	.0027	.0036	.0047	.0064	.0095	.016
25	.0009	.0013	.0017	.0023	.0030	.0039	.0052	.0072	.011	.020
30	.0010	.0014	.0019	.0025	.0033	.0043	.0057	.0082	.013	
40	.0013	.0017	.0023	.0030	.0039	.0052	.0072	.011	.020	
50	.0016	.0021	.0027	.0036	.0047	.0064	.0095	.016		
55	.0017	.0023	.0030	.0039	.0052	.0072	.011	.020		
60	.0019	.0025	.0033	.0043	.0057	.0082	.013			
65	.0021	.0027	.0036	.0047	.0064	.0095	.016			
70	.0023	.0030	.0039	.0052	.0072	.011	.020			
75	.0025	.0033	.0043	.0057	.0082	.013				
80	.0027	.0036	.0047	.0064	.0095	.016				
85	.0030	.0039	.0052	.0072	.011	.020				
90	.0033	.0043	.0057	.0082	.013					
95	.0036	.0047	.0064	.0095	.016					
100	.0039	.0052	.0072	.011	.020					
105	.0043	.0057	.0082	.013						
110	.0047	.0064	.0095	.016						
115	.0052	.0072	.011	.020						
120	.0057	.0082	.013							
125	.0064	.0095	.016							
130	.0072	.011	.020							
135	.0082	.013								
140	.0095	.016								
145	.011	.020								
150	.013									
155	.016									
160	.020									

Table (2.15). Base failure rate of diodes.

T (°C)	S .1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.0006	.0009	.0013	.0018	.0024	.0033	.0044	.0059	.0081	.011
10	.0008	.0012	.0016	.0022	.0030	.0039	.0053	.0072	.010	.014
20	.0010	.0015	.0020	.0027	.0036	.0048	.0065	.0090	.012	.019
25	.0012	.0016	.0022	.0030	.0039	.0053	.0072	.010	.014	.022
30	.0013	.0018	.0024	.0033	.0044	.0059	.0081	.011	.017	
40	.0016	.0022	.0030	.0039	.0053	.0072	.010	.014	.022	
50	.0020	.0027	.0036	.0048	.0065	.0090	.012	.019		
55	.0022	.0030	.0039	.0053	.0072	.010	.014	.022		
60	.0024	.0033	.0044	.0059	.0081	.011	.017			
65	.0027	.0036	.0048	.0065	.0090	.012	.019			
70	.0030	.0039	.0053	.0072	.010	.014	.022			
75	.0033	.0044	.0059	.0081	.011	.017				
80	.0036	.0048	.0065	.0090	.012	.019				
85	.0039	.0053	.0072	.010	.014	.022				
90	.0044	.0059	.0081	.011	.017					
95	.0048	.0065	.0090	.012	.019					
100	.0053	.0072	.010	.014	.022					
105	.0059	.0081	.011	.017						
110	.0065	.0090	.012	.019						
115	.0072	.010	.014	.022						
120	.0081	.011	.017							
125	.0090	.012	.019							
130	.010	.014	.022							
135	.011	.017								
140	.012	.019								
145	.014	.022								
150	.017									
155	.019									
160	.022									

Table (2.16). Base failure rate of thyristors.

2.6 Lasers

A laser is a device which transforms incoherent light of various frequencies of vibration into a very narrow, intense beam of coherent light. The name is derived from the initial letters of "Light Amplification by Stimulated Emission of Radiation". Some types of lasers include Helium/Neon lasers, Argon Ion lasers and Solid state Ruby rod lasers.

The models and failure rates presented in this section apply to the laser peculiar items only, i.e. those items wherein the lasing action is generated and controlled. These actions are the lasing media, the laser pumping mechanism and the coupling method. Thus the general laser failure rate model is:

$$\lambda_{LASER} = \lambda_{MEDIA} + \lambda_{PUMP} + \lambda_{COUPLING} \tag{2.9}$$

Examples of media related hardware and influence factors are the solid state rod, the gas, the vacuum integrity and the tube diameter. The coupling function contributors are the mirrors, the substrates and the level of dust protection provided. In the following subsection the failure rate of all mentioned laser types will be studied.

2.6.1 Helium/Neon and Argon Ion Lasers

The failure rate of Helium/Neon lasers is given as:

$$\lambda_{He/Ne} = \pi_E \;\; MEDIA + \pi_E \;\; COUPLING \tag{2.10}$$

where, $\lambda_{He/Ne}$ is the failure rate of the Helium/Neon laser in failures per 10^6 hours,

π_E is the environmental application factor as defined in table (2.2),

λ_{MEDIA} is the failure rate contribution of the lasing media and its value is 84 failures per 10^6 operating hours,

$\lambda_{COUPLING}$ is the failure rate contributions of the lasing coupling hardware and its value is 0.1 failures per 10^6 hours of operation.

It is obvious from the above that a more simplified from of the failure rate can be obtained as:

$$\lambda_{He/Ne} = 84.1 \; \pi_E \tag{2.11}$$

In a similar manner the failure rate of Argon Ion lasers can be approximated to:

$$\lambda_{AI} = 463 \; \pi_E \tag{2.12}$$

where, λ_{AI} is the failure rate of the Argon Ion laser in failures per 10^6 hours of operation, and

π_E is the environmental application factor.

2.6.2 The Failure Rate of Solid State Ruby Rod Lasers

The failure rate for solid state ruby rod lasers is given as:

$$\lambda_{RUBY} = \pi_E \lambda_{MEDIA} + \lambda_{PUMP} + 16.3 \pi_E \pi_C \pi_{OS} \tag{2.13}$$

where, π_E is the environmental application factor as defined in table (2.2),

λ_{MEDIA} is the failure rate contributions of the lasing material in failures per 10^6 hours of operation,

λ_{MEDIA} is calculated using the empirical formula:

$$\lambda_{MEDIA} = (\pi_{REP}) (43.5 \quad F^{2.52}) \tag{2.14}$$

(failures per 10^6 hours)

where, π_{REP} is the rate factor as defined in section (2.2)

F is the energy density in joules per square centimetres per pulse over the cross sectional area of the laser beam.

π_C is the cleanliness factor as defined in section (2.2),

π_{OS} is the optical surface factor and is equal to the active optical surfaces,

λ_{PUMP} is the failure rate contributions of the pumping mechanism and its value for Krypton lamp is calculated from the following equation:

$$\lambda_{PUMP} = \pi_E \left((625) (10^{(2.3\ P/L)}) \pi_{COOL} \right) \qquad (2.15)$$

(failures per 10^6 hours)

where, P is the average input power in kW. (a typical value is 4 kW),

L is the flashlamp or flashtube arc length in cm,

and π_{COOL} is the cooling factor as defined in section(2.2).

In order to obtain a rough figure of the failure rate of solid state ruby rod lasers, consider the following example.

A solid state ruby rod laser is used in a naval sheltered environment and has the following characteristics. The flashlamp is Krypton type and is cooled by liquid-fluorocarbon. The pulse rate is 1 pulse per second and the energy density is 10 joules per square centimetre per pulse.

The average input power is 2 kW and the length of the flashlamp arc is 5.08 cm. The area of the rod is 50 cm^2. The equipment is treated by trained personnel and its cleaning is quite rigorous. The total number of optical surfaces is 7. It is required to find the failure rate of the laser and its reliability for one day and also for one year period of continuous operation.

From table (2.2) π_E for naval sheltered environment is equal to 5. From table (2.10) π_{REP} for a rate of 1 pulse per second is 3,600. From section (2.2), the cleanliness factor π_C is 1.0 and from table (2.9), the flashlamp cooling factor π_{COOL} for liquid-fluorocarbon is 0.1. Substituting the relevant quantities in eq.(2.16),

$$\lambda_{MEDIA} = (3{,}600)\ (43.5 \times 0.2^{(2.52)})$$

$$= .2712 \ \text{(failures per } 10^6 \text{ hours)} \qquad (2.16)$$

similarly,

$$\lambda_{PUMP} = 5\ (\ 625\ (10)^{.9} \times 0.1\ )$$

$$= 5 \times 625 \times 7.94 \times 0.1$$

$$= 2{,}482 \quad \text{(failures per } 10^6 \text{ hours)} \qquad (2.17)$$

Finally,

$$16.3 \times \pi_E \times \pi_{OS} \times \pi_C = 16.3 \times 5 \times 7 \times 1$$

$$\doteq 570 \text{ (failures per } 10^6 \text{ hours)} \qquad (2.18)$$

Therefore the total failure rate λ_p is:

$$\lambda_p = 2{,}482 + 570 + 2712 = 5.764 \text{ (failures per } 10^6 \text{ hours)} \qquad (2.19)$$

The reliability of the laser for a time period of 24 hours is:

$$R = e^{-\lambda t} = \exp(-\frac{5764}{10^6} \times 24)$$

$$= e^{-.1383} = 0.8708$$

and for one year period, the reliability is:

$$R = \exp(-\frac{5764}{10^6} \times 8760) = e^{-50.5} \doteq 0$$

which implies that the event of having at least one failure per year of operation is practically certain.

2.7 Reliability of Microelectronic Devices

This section presents failure rate prediction models for two major classes of microelectronic devices, namely, the Monolithic Bipolar & MOS Digital (SSI/MSI), and the Monolithic Bipolar & MOS Memories. In the title description of each monolithic device type, SSI and MSI represent Small Scale Integration and Medium Scale Integration respectively. MOS represents all metal-oxide semiconductor mocrocircuits which include NMOS, PMOS, CMOS and MNOS fabricated on various substrates, such as sapphire, polycrystalline or single crystal silicon.

The general expression for the failure rate model for microelectronic has as follows:

$$\lambda_p = \lambda_m + \lambda_T \tag{2.20}$$

where, λ_p is the overall device failure rate for monolithic devices,

λ_m is the failure rate component due to mechanical (application, environment) causes, and

λ_T is the failure rate component due to time degradation causes, and represnets degradation mechanisms which are accelerated by temperature and electrical bias.

2.7.1 Monolithic Bipolar and MOS Digital SSI/MSI Devices

The general failure rate formula for these devices is:

$$\lambda_p = \pi_L \pi_Q (C_1 \pi_T + C_2 \pi_E) \pi_p \tag{2.21}$$

where, λ_p is the device failure rate in failures per 10^6 hours,

π_L is the device learning factor. The value of this factor is 10 if:

a) the device is in the initial production stage,

b) there have been major changes in design or process, and

c) if the production line personnel has been radically changed.

π_L is equal to 1.0 for all production conditions not listed above,

π_Q is the quality factor as defined in table (2.3),

π_E is the environmental factor also defined in table (2.2),

π_T is the temperature accelerator factor and its values are given in table (2.17),

C_1 and C_2 are the circuit complexity failure rates and are equal to:

$C_1 = 0.00129\ (G)\ (0.677)$ (failures per 10^6 hours)

$C_2 = 0.00389\ (G)\ (0.359)$ (failures per 10^6 hours)

where, G is the number of gates, and finally

π_p is the pin factor given in table (2.18).

T_j (°C.)	π_T	T_j (°C.)	π_T	T_j (°C.)	π_T
25	.10	51	.36	77	1.1
27	.11	53	.40	79	1.2
29	.12	55	.44	81	1.3
31	.14	57	.48	83	1.4
33	.15	59	.52	85	1.5
35	.17	61	.57	87	1.6
37	.19	63	.62	89	1.7
39	.21	65	.67	91	1.8
41	.23	67	.73	93	2.0
43	.25	69	.79	95	2.1
45	.28	71	.86	97	2.3
47	.30	73	.96	99	2.5
49	.33	75	1.0	101	2.6

Table (2.17). π_T against junction temperature.

No. of Pins	π_p
<24	1.0
24 to 40	1.1
42 to 64	1.2
>64	1.3

Table (2.18). The pin factor π_p.

2.7.2 Monolithic MOS and Bipolar Memories

The failure rate for these devices is given as:

$$\lambda_p = \pi_L \pi_Q (C_1 \pi_T + C_2 \pi_E) \pi_p \quad (2.22)$$

(failures per 10^6 hours)

where again, λ_p, π_L, π_E, π_Q, and π_T have the same values as in the case of Monolithic Bipolar and MOS Digital SSI / MSI. The complexity factors C_1 and C_2 however, are different and for RAMs (Random Access Memories) are:

$C_1 = 0.00199 \; B^{0.603}$ failures per 10^6 hours,

$C_2 = 0.00056 \; B^{0.644}$ failures per 10^6 hours,

where, B is the number of bits,

and finally for ROMs (read only memories),

$C_1 = 0.00114 \; B^{.603}$ failures per 10^6 hours,

$C_2 = 0.00032 \; B^{.646}$ failures per 10^6 hours.

This concludes the study of the reliability of electronic systems. It must be emphasized at this point, that the part failure rate method is the only acceptable method in military standanrds and is one of its kind.

CHAPTER III

RELIABILITY OF POWER DISTRIBUTION SYSTEMS

3.1 Introduction

The scope of this chapter is to illustrate techniques used to determine the reliability of power distribution systems.[13-15]

The chapter begins by introducting the failure-repair cycle, the cycle time and the availability of power distribution systems. Then the repairable systems are considered. The average time to repair r and the average time failure m are defined for both series and parallel configurations.

Models for redundant component overlapping outages follow. They include maintenance effects, failure bunching, overload effects and common-mode failures.

The chapter concludes with a study of substation reliability prediction. This introduces five steps to be followed when predicting the reliability of a substation, an example application follows and finally two industrial substations are studied and their reliabilities compared.

3.2 The Failure-repair Cycle, Cycle Time and Availability

A power distribution system is expected to have continuous operation. However failures due to many reasons do happen. Consequently a failure-repair cycle study is necessary.

To formulate the failure-repair cycle, a power distribution system is observed for an interval of time (t_1, t_2) in which N cycles of failure and repair are noted.

Let the observed time-to-failure for the first cycle be m_1 and the time-to-repair observed for that failure be r_1. Similarly, let m_i and r_i be the observed times to failure and repair for the ith cycle. The average cycle of the failure-repair process $\overline{T}$ then is given by the sum of the average time-to-failure and the average time to repair, or:

$$\overline{T} = \overline{m} + \overline{r} \tag{3.1}$$

where,

$$\overline{m} = \frac{1}{N} \sum_{1}^{N} m_i \tag{3.2}$$

and

$$\overline{r} = \frac{1}{N} \sum_{1}^{N} r_i \tag{3.3}$$

The time period that the system is available for service is given by the ratio of average uptime $\bar{m}$ to average cycle time $\bar{T}$ and the availability A is:

$$A = \frac{\bar{m}}{\bar{T}} = \frac{\bar{m}}{\bar{m} + \bar{r}} \tag{3.4}$$

The unavailability, defined as $\bar{A} = 1 - A$, is then given by the ratio of the average downtime $\bar{r}$ to the average cycle time $\bar{T}$:

$$\bar{A} = 1 - A = \frac{\bar{r}}{\bar{T}} = \frac{\bar{r}}{\bar{m} + \bar{r}} \tag{3.5}$$

The fault frequency f is defined as the reciprocal of the average cycle time and consequently is given by:

$$f = \frac{1}{\bar{T}} = \frac{1}{\bar{m} + \bar{r}} \tag{3.6}$$

Fig.(3.1) shows a "two-state model" from which the above formulas are obtained.

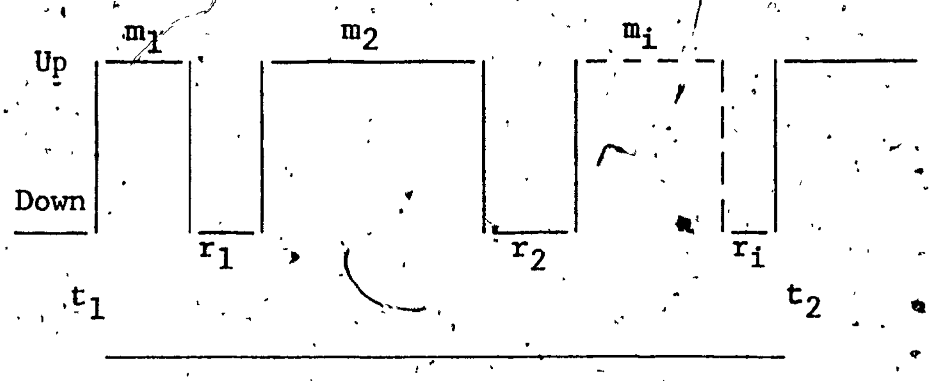

Fig.(3.1). A two-state model undergoing i failures and repairs.

3.3 Repairable component in series

A system is said to be connected in series if each component can be regarded as a properly working switch. Fig.(3.2) shows two components of a distribution system connected in series. Both are required to be in service at the same time. Assume each is governed by a stationary renewal process with average times to failure of m_1 and m_2 and times to repair of r_1 and r_2.

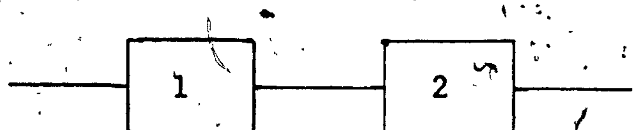

Fig.(3.2) Two series systems

Finally let both components be statistically independent. Then, if A_1 is the availability of component 1 and A_2 is the availability of component 2, the system availability A_s is given by:

$$A_s = A_1 \cdot A_2 \tag{3.7}$$

or, from eq.(3.4):

$$A_s = \frac{m_1}{m_1 + r_1} \cdot \frac{m_2}{m_2 + r_2} \tag{3.8}$$

An equivalent component operating as the combination of components 1 and 2 would have an availability:

$$A_s = \frac{m_s}{m_s + r_s} \tag{3.9}$$

where m_s and r_s are the average uptime and down time respectively for this equivalent component.

The frequency of the equivalent system failure is equal to the sum of the average frequency of the events of component 1 failing while 2 is operating plus the frequency of event of component 2 failing while 1 is operating.

$$f_s = A_2 \cdot f_1 + A_1 \cdot f_2 = \frac{1}{m_s + r_s}$$

$$= \frac{m_2}{m_2 + r_2} \left(\frac{1}{m_1 + r_1} \right) + \frac{m_1}{m_1 + r_1} \left(\frac{1}{m_2 + r_2} \right) \qquad (3.10)$$

In order to express m_s and r_s as a linear combination of r_1, r_2 and m_1, m_2 consider the following. When one of the components has failed, say component 1, the system is down and consequently a failure of component 2 must be counted as a failure. Therefore from eq.(3.9)-(3.10) the average up duration m_s is given by:

$$m_s = \frac{A_s}{f_s} = \frac{m_2 \, m_1}{m_1 + m_2} \qquad (3.11)$$

or,

$$\frac{1}{m_s} = \frac{1}{m_1} + \frac{1}{m_2} \qquad (3.12)$$

But $1/m_s$ by definition is equal to the failure rate λ_s. Therefore eq.(3.12) becomes:

$$\lambda_s = \lambda_1 + \lambda_2 \qquad (3.13)$$

The equivalent repair time r_s may now be solved by substituting m_s from eq.(3.12) into eq.(3.7) and eq.(3.8).

$$r_s = \frac{1 - A_s}{f_s} \tag{3.14}$$

$$= \frac{(m_1 + r_1)\ (m_2 + r_2) - m_1 m_2}{m_1 + m_2} \tag{3.15}$$

$$= \frac{\dfrac{r_1}{m_1} + \dfrac{r_2}{m_2} + \left(\dfrac{r_1}{m_1}\right)\left(\dfrac{r_2}{m_2}\right)}{\dfrac{1}{m_1} + \dfrac{1}{m_2}}$$

$$= \frac{\lambda_1 r_1 + \lambda_2 r_2 + (\lambda_1 r_1)\ (\lambda_2 r_2)}{\lambda_1 + \lambda_2} \tag{3.16}$$

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + (\lambda_1 r_1)\ (\lambda_2 r_2)}{\lambda_s} \tag{3.17}$$

Note that if one component cannot fail while the other is on repair, eq.(3.17) becomes:

$$r_s = \frac{\lambda_1\ r_1 + \lambda_2\ r_2}{\lambda_s} \tag{3.18}$$

In practice the product $\lambda \cdot r$ is generally less than 0.1 for generation equipment and generally less than 0.01

for all transmission and distribution equipment. To appreciate the above, consider the following example.

A feeder consists of a 3 km overhead section with a failure rate λ_1 = 0.06 fault per circuit kilometre year, and a 2.5 km underground section with failure rate, λ_2 = 0.06 fault per circuit kilometre. The cable has two terminations with a failure rate, λ_3 = 0.003 fault per termination year.

The repair times for the three components are 5, 20 and 5 hours respectively for the overhead section, the underground section and the cable terminations. It is required to find the failure rate and restoration time for the feeder.

The example given below is an application to power distribution systems. However, it can very well apply to other systems (civil, mechanical) provided they are repairable and the required quantities are available. The calculation is given in the following table (3.1).

Component	λ (per year)	r(h)	λr (hour per year)	λr (year per year)
Overhead section	0.18	5	0.9	0.000102
Underground section	0.15	20	3.0	0.000342
Cable terminations	0.006	5	0.03	0.00000342
Contribution to feeder	0,336		3.93	0.000447

$$r_s = \frac{3.93}{0.336} = 11.69 \text{ hours.}$$

$$\lambda_s = 0.336 \text{ per year.}$$

$$\overline{A}_s = 0.000477$$

$$A = 0.9995525$$

Table (3.1). Calculations for the three km feeder example

3.4 Repairable components in parallel

A power distribution system with two repairable components in parallel will be studied. Again it can be any other system provided that the failure and repair process are stationary such that the distributions of down times and up times form a renewal process. The system is down when both components are down; that is the outages of the two overlap. See fig. (3.3)

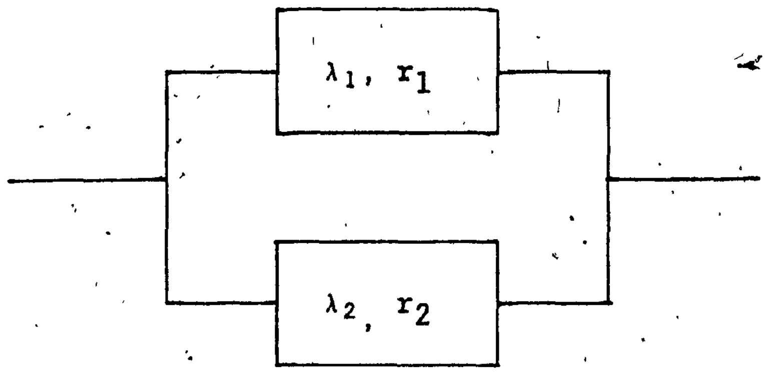

Fig. (3.3) Parallel-connected systems.

The unavailability, the frequency failure, the average up time m_p and the average down time r_p of an equivalent element will be determined. First the unavailability $\overline{A}_p$ is given by the product of the unavailability of component 1, $\dfrac{\lambda_1 r_1}{(1 + \lambda_1 r_1)}$ and component 2, $\dfrac{\lambda_2 r_2}{(1 + \lambda_2 r_2)}$

$$\overline{A} = \frac{(\lambda_1 r_1)\ (\lambda_2 r_2)}{(1 + \lambda_2 r_2)\ (1 + \lambda_1 r_1)} \tag{3.19}$$

The frequency of system failure is given by:

$$f_p = f_1 \bar{A}_2 + f_2 \bar{A}_1$$

and,

$$f_p = \left(\frac{\lambda_1}{1 + \lambda_1 r_1} \right) \left(\frac{\lambda_1 r_2}{1 + \lambda_2 r_2} \right) + \left(\frac{\lambda_2}{1 + \lambda_2 r_2} \right) \left(\frac{\lambda_1 r_1}{1 + \lambda_1 r_1} \right) \tag{3.20}$$

$$f_p = \frac{\lambda_1 \lambda_2 \ (r_1 + r_2)}{(1 + \lambda_1 r_1) \ (1 + \lambda_2 r_2)} \tag{3.21}$$

If the system is replaced by an equivalent element the equivalent element would have an average up time m_p and and an average down time r_p. From eq.(3.5) and eq.(3.6) this r_p can be expressed in terms of $\bar{A}_p$ and f_p as follows:

$$r_p = \frac{\bar{A}_p}{f_p} = \frac{r_1 \ r_2}{r_1 + r_2} \tag{3.21}$$

and from eq.(3.4), eq.(3.19) and eq.(3.21) m_p can be determined by substitution:

$$m_p = \frac{1 + \lambda_1 r_1 + \lambda_2 r_2}{(\lambda_1 \ \lambda_2) \ (r_1 + r_2)} \tag{3.22}$$

and since the failure rate is by definition the reciprocal of the up time of the system; the failure rate of the equivalent element if given by:

$$\lambda_p = \frac{1}{m_p} = \frac{\lambda_1 \lambda_2 \, (r_1 + r_2)}{1 + \lambda_1 r_1 + \lambda_2 r_2} \qquad (3.23)$$

3.5 Overlapping outage models for redundant components

Overlapping outages are defined as outages occuring at the same time as one or more of the system's components are down. They can be categorized as forced outages during maintenance periods, failure bunching due to environmental effects, overload induced outages of a second component during the forced outage of the first component, and common-mode failure.

In the following four sections each of these will be studied.

3.5.1 Maintenance effects

Fig.(3.4) shows the schematic representation of a forced outage occuring during maintenance time. The assumption is that during a maintenance outage of one of two redundant components it is possible that the second component may suffer an unscheduled or forced outage.

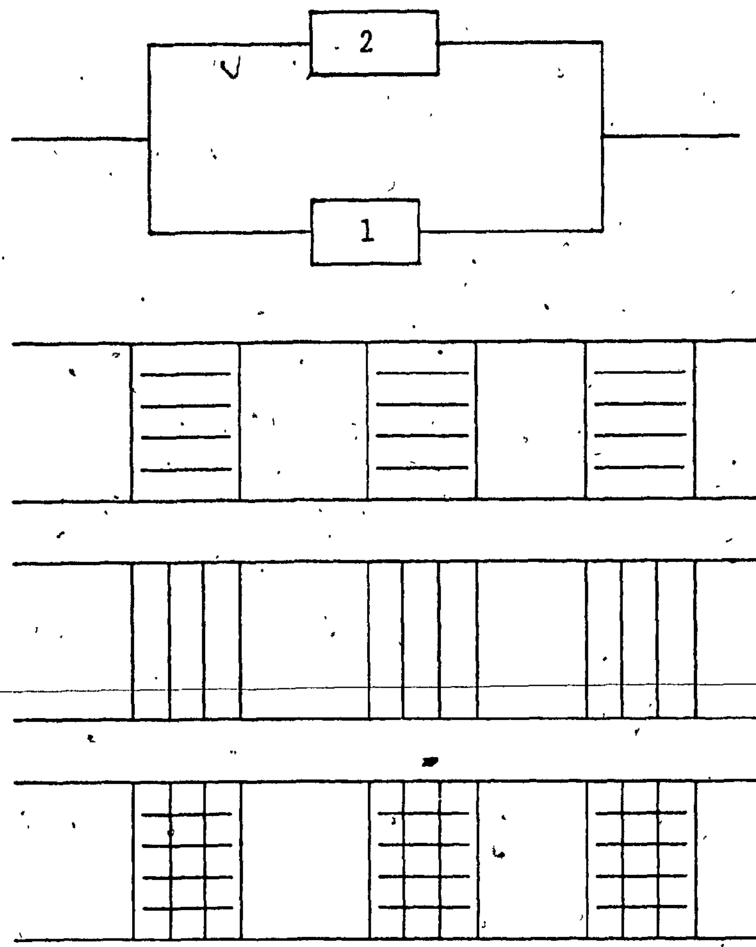

Fig.(3.4) Overlapping outages in two parallel redundant systems.

Suppose that component 2 is undergoing maintenance E_2 times per unit time and forced outages events on the first component occur f_1 times per year. Finally letting λ_i, r_i and f_i be the average failure rate, repair time and outage frequency for component number 1 and if r_m is the average duration of maintenance outage for component 2, then the forced outage overlapping maintenance has a rate of occurrence of:

$$\frac{1 + \lambda_1 r_1}{1 + \cfrac{\lambda_1 r_1}{1 + \cfrac{r_1}{r_m}}} \cdot f_1 \cdot E_2 \qquad (3.24)$$

Note that the rate of occurrence is calculated in times per year.

3.5.2 Failure Bunching

Failure bunching is defined as the risk of overlapping forced outage during periods of high environmental stress. Fig. (3.5) illustrates periods of severe weather labeled S followed by periods of normal weather labeled N. This implies that severe weather periods recur on the average once in N + S hours and persist for S hours.

The failure rate during normal weather is λ and λ' during severe weather. It is obvious that $\lambda' > \lambda$. The ratio λ' / λ is usually designated by p. The lower part of Fig. (3.4) shows a device which consists of two elements in parallel, say two transformers, and their equivalent.

The average failure for the device is called λ_e and is given by eq.(3.25),

$$\lambda_e = \frac{(S\lambda' + N\lambda)}{N + S} = \frac{(pS + N)\,\lambda}{N + S} \tag{3.25}$$

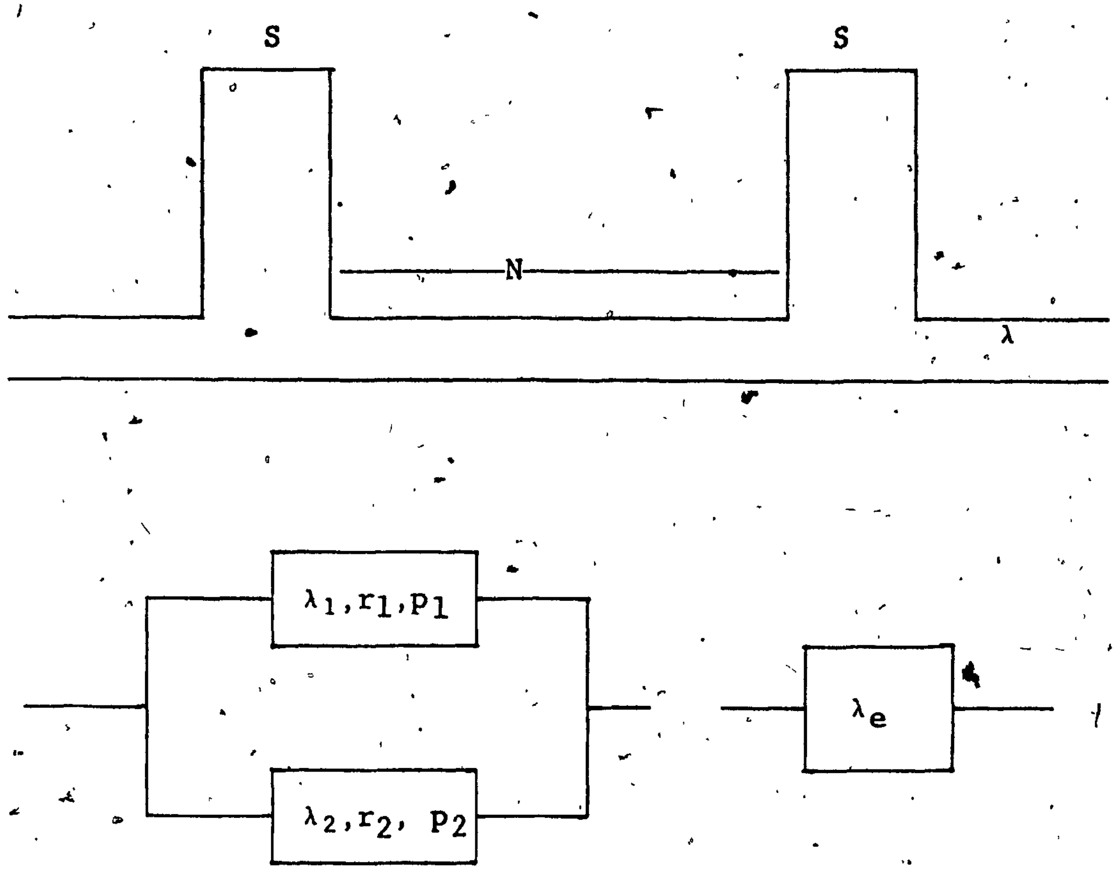

Fig. (3.5) Failure bunching

When dealing with redundant systems however a different approach is used. To illustrate this approach, let r_1 and r_2 be the average repair times for transformers 1 and 2 for normal weather. Let λ_1, λ_2 be the average failure rates for 1 and 2 respectively for normal weather too, and finally let p_1 and p_2 be the ratios of severe weather failure ratio to normal weather failure rate for 1 and 2 respectively. Then the rate of overlapping failure for two transformers subject to normal and severe weather is given by:

$$\lambda_p = \frac{N}{N + S} \lambda_1 \lambda_2 \left((r_1 + r_2) \left(1 + \frac{S}{N} (p_1 + p_2) \right) + 2 \frac{S^2}{N} p_1 p_2 \right) \qquad (3.26)$$

3.5.3 Overload Effects

Failure overlapping in a power distribution system due to overload may occur during forced outage or maintenance outage of other component of that system.

Fig.(3.6) illustrates two transformer banks and overload contingency curves for a transformer. Suppose that one transformer bank is out of service for a period of time, then there is a risk that the load will exceed the capability of the remaining bank. The risk depends on the seasonal load and the ambient temperature. The curves of fig.(3.6) were developed for northeast U.S.A. seasonal load types and ambient temperatures; and they illustrate that risk of overload.

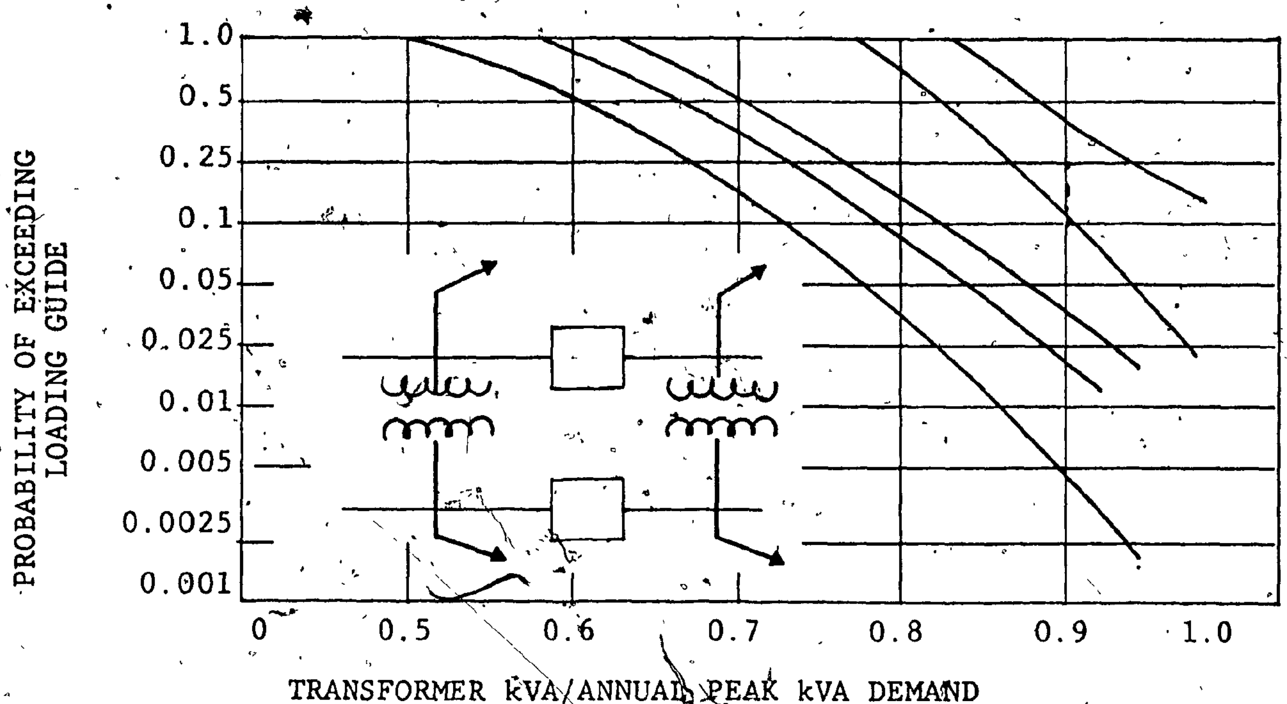

Fig.(3.6) Overload contingency curves for a transformer bank

To calculate the overload event rate, three factors are involved: first, the unscheduled outage rate for transformers λ_T; second, the distribution of outage duration for transformers $P_r(R)$ and finally the probability that an overload will result given the outage duration, $P(O|R)$. The overload event rate f_o is given by the product of the above three quantities. i.e.

$$f_o = \lambda_T \cdot P(R) \cdot P(O|R) \tag{3.27}$$

3.5.4 Common-mode failures

Common-mode failures are defined as systematic non-random events causing an outage of all the redundant components of a system. Some examples include cases of components with the same built-in design error, dependance of all redundant components on a single common element, systematic human error and changes in the environment.

Very little data is available on the rate with which common-mode events have occured. This is why there is no mathematical model to describe the phenomenon. The fact that failures of this type have occured however, is sufficient to recommend extreme caution when designing and testing the system. To appreciate this, consider the case of the protective and control apparatus of a nuclear reactor in which a common-mode failure would have catastrophic results.

3.5.5 Reliability procedures for substations

The evaluation of the reliability of a substation can be done in five steps. The first step is the physical system description. Here the components and their reliability indexes are specified. The second step is to specify performance criteria. This includes voltage, current and frequency quantities required by the customer for continuous service. The third step is to set a reliability goal. This means that the designer is required to meet certain quality standards. The fourth step is an analysis of the failure modes and their effects. This analysis studies the component failure patterns. This includes their contingency levels, the sequence of occurence and the effect on the remaining circuit. The final step is called accumulation of failure effects. The role of this final step is to calculate the final value of the availability of the system and compare it with the goal value that is being set in step three.

To illustrate the use of the above, consider the following example-application. Fig.(3.7) shows a ring-bus configuation. Lines L_1 and L_2 are far apart so that supply and capability are redundant. Lines S_1 and S_2 feed a subtransmission network. The rating of each transformer is two-thirds (2/3) the annual peak load of the station. The outage data are given in table (3.2)

To conclude the physical description of the system, let the failure events of the switches be assigned to the bus, breaker or transformer switched by the device.

	Failure Rate (per year)	Outage Duration (h)	Unavail- ability	Probabi- lity
Circuit Breakers				
Circuit breaker fault (backup required)	0.007	72	-	-
Maintenance	-	16	0.004	-
Probability of breaker found inoperative	-	-	-	0.0005
Transformer				
Forced outage	0.0012	168	-	-
Maintenance	-	12	0.004	-
Bus Section				
Forced Outage	0.007	3.5	-	-
Maintenance (combined with line)	-	18	0.003	-
Line Section				
Forced Outage	0.05	23	-	-
Maintenance	-	15	0.005	-

Table (3.2). Component outage data for the ring-bus example.

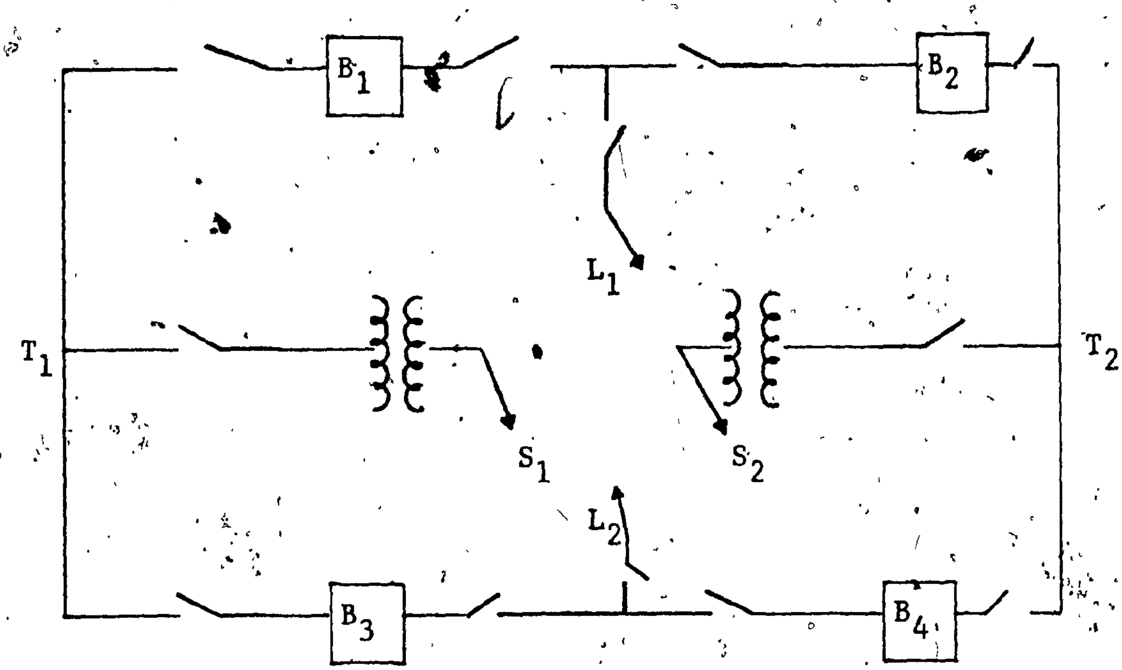

Fig. (3.7) Ring-bus example.

The system is operating satisfactorily as long as one of the two lines is feeding the subtransmission network, and as long as the transformer is not overloaded. These criteria take care of the second step, that is, the performance criteria. No goal was set for this example. This is generally the case with power systems. The designer predicts reliability levels instead of planning for it. This however is not the case with the design of spacecrafts, nuclear reactors and military equipment, where reliability standards are to be met in order to avoid catastrophies.

The fourth step is an analysis of failure modes and effects. Fig. (3.8) shows the ring-bus configuration with breaker B_2 stuck, and breaker B_1 suffering a failure. It is obvious that the effect of this failure mode would be a substation outage with duration equal to the switching operation.

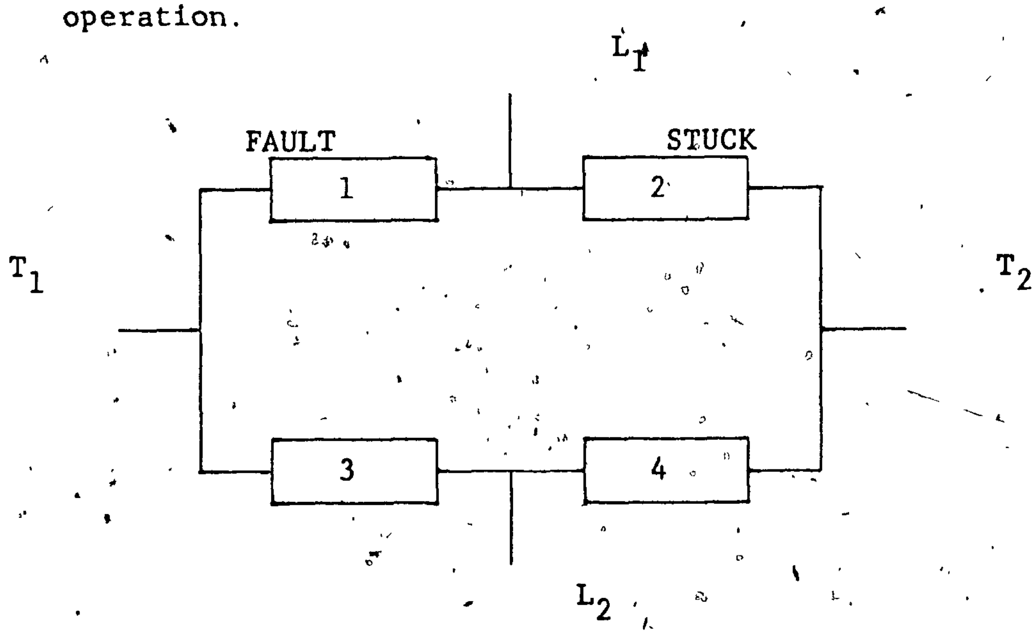

Fig. (3.8) Ring-bus with stuck breaker.

Table (3.3) shows all possible failure events and their effects on the operation of the substation. The first column of table (3.3) is a list of all possible failure events. It is followed by a column labeled combinations. This column denotes the number of possible combinations in which the failure event described in column one, can occur. Column number three describes the conditional probability of the event. The next column is labeled restoration means and it denotes the means of restoration when an outage has occured. In this example normal operation is restored by switching or by repair. They are respectively assigned as S and R. Following is the duration column indicating the duration of the fault. The sixth column is the frequency per 1,000 years column, indicating the number of failures in a time period of 1,000 years. Columns two to six are called event data and are taken from practice experience and manufacturer's specifications.

The last three columns are called station effect. The first of the three is labeled frequency per 1,000 years. It indicates the number of the station outages in a period of time of 1,000 years. It is calculated by multiplying the failure rate with the conditional probability and the number of combinations. The second column indicates the total number of hours of outage in 1,000 years. This value is found by multiplying the frequency per 1,000 years with the duration in hours. The last column ranks the outages with respect to their frequency of occurence in 1,000 years time period.

Summing all the values listed under frequency per 1,000 years, the station interruption per 1,000 is obtained. In this example the value is 1.16 per 1,000 years.

Failure Event	Event Data				Station Effect		
	Combinations (1)	Exposure (2)	Restoration Means (3)	Duration (h) (4)	Freq. (per 1000 years) (5)	Freq. (per 1000 years) (6)	H (hours per 1000 years) (7)
I. Brkr. Fault per Stuck Brkr.	8	~ 1	S*	1	0.0035	0.028	0.028
II. Forced Outage per Maintenance on another Component							
A. Line and Line Bus Maintenance Line F.O. per maint.	2	0.005	R	9.1	50	0.5	4.52
Line bus F.O. per maint.	2	0.005	R	2.8	7	0.07	0.196
Brkr. fault per maint.	4	0.005	S	1	7	0.14	0.14
B. Transf. and Bus Maintenance							
Trans. F.O. per maint.	2	0.004	R	11.2	12	0.096	1.07
Trans. bus F.O. per maintenance	2	0.004	R	2.7	7	0.056	0.152
Brkr. fault per maint.	4	0.004	S	1	7	0.112	0.112
C. Brkr. Fault per Brkr. Maint.	4	0.004	S	1	0.112	0.112	0.112

cont'd

Failure Event	Event Data				Station Effect		
	Combinations (1)	Exposure (2)	Restoration Means (3)	Duration (h) (4)	Freq. (per 1000 years) (5)	Freq. (per 1000 years) (6)	H (hours per 1000 years) (7)
III. Overlapping Independent Forced Outages Outages							
(trans. + Bus)	2	~ 1	R	53.5	0.0044	0.0089	0.475
(Line + Bus)	2	~ 1	R	10.3	0.0076	0.0152	0.156
Breaker Opposite breakers	4	~ 1	S	1	0.0004	0.0016	0.0016
Breaker Adjacent breakers	8	~ 1	S	1	0.0000 056	0.000 04	0.00004
Breaker · Transformer	8	~ 1	S	1	0.0016	0.0129	0.0129
Breaker · Line	8	~ 1	S	1	0.00091	0.0073	0.0073
Bus · Breaker	16	~ 1	S	1	0.00002	0.0003	0.0003
	Station Interruption:					1.16	7.0
	Average Duration = 6.0 h						

* Restoration by switching
! Restoration by repair
Brkr. = Breaker

F.O. = Forced Outage
Maint. = Maintenance
Transf. = Transformer

Table (3.3). Failure events of the equipment of the substation and their effects on the operation of the substation.

Also summing the hours per 1,000 years the total time of station failure is obtained. In this example it is found to be 7.0 hours per 1,000 years. The average duration then is 7/1.16 = 6.03 hours.

The final step for the complete analysis of this example is the accumulation of failure effects. So far only station interruption has been studied. Component overload is another effect that should be studied. Once this has been done then a composite value or reliability can be obtained.

3.5.6 Industrial substation example

The following example compares the reliability of two different industrial substations. Figures (3.9) and (3.10) show the two one-line diagrams of the substations. Fig. (3.9) illustrates a 46-kV single source feeding a single-low-voltage-bus of 13.8 kV. Fig. (3.10) illustrates two 46-kV sources feeding a single-low-voltage-bus of 13.8 kV. It is obvious that the second apparatus is more expensive than the first because it has the following extra components.

a) one 46-kV feed,

b) three 46-kV breakers and

c) two 13.8-kV breakers.

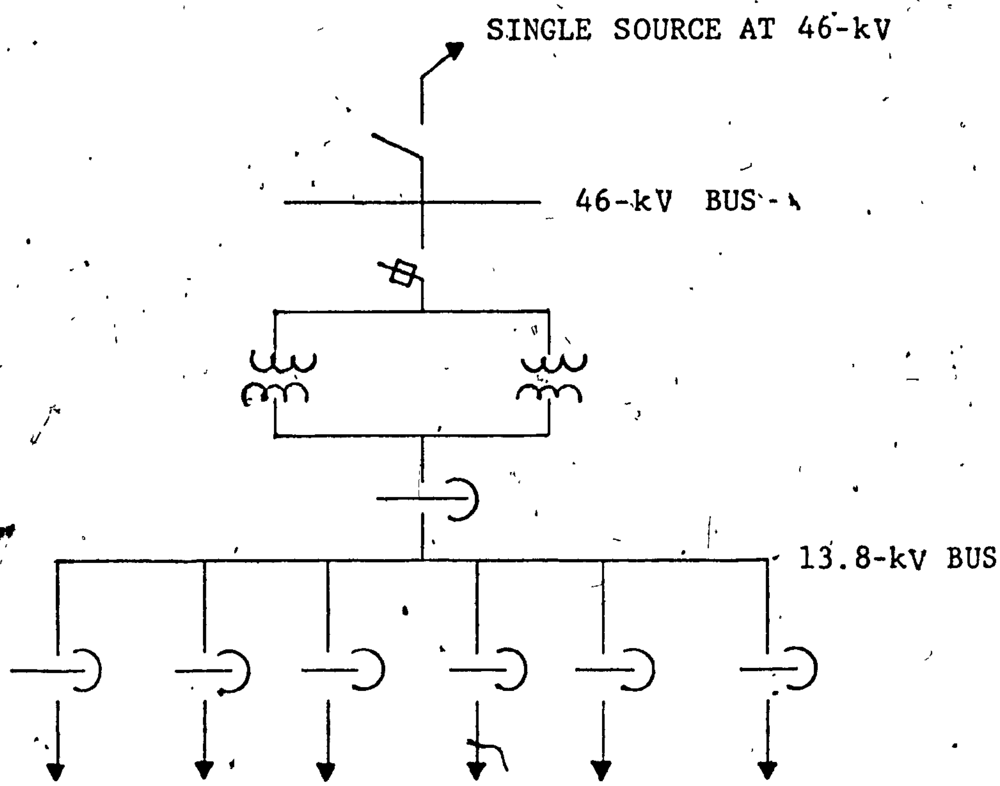

Fig.(3.9) Industrial substation with one source.

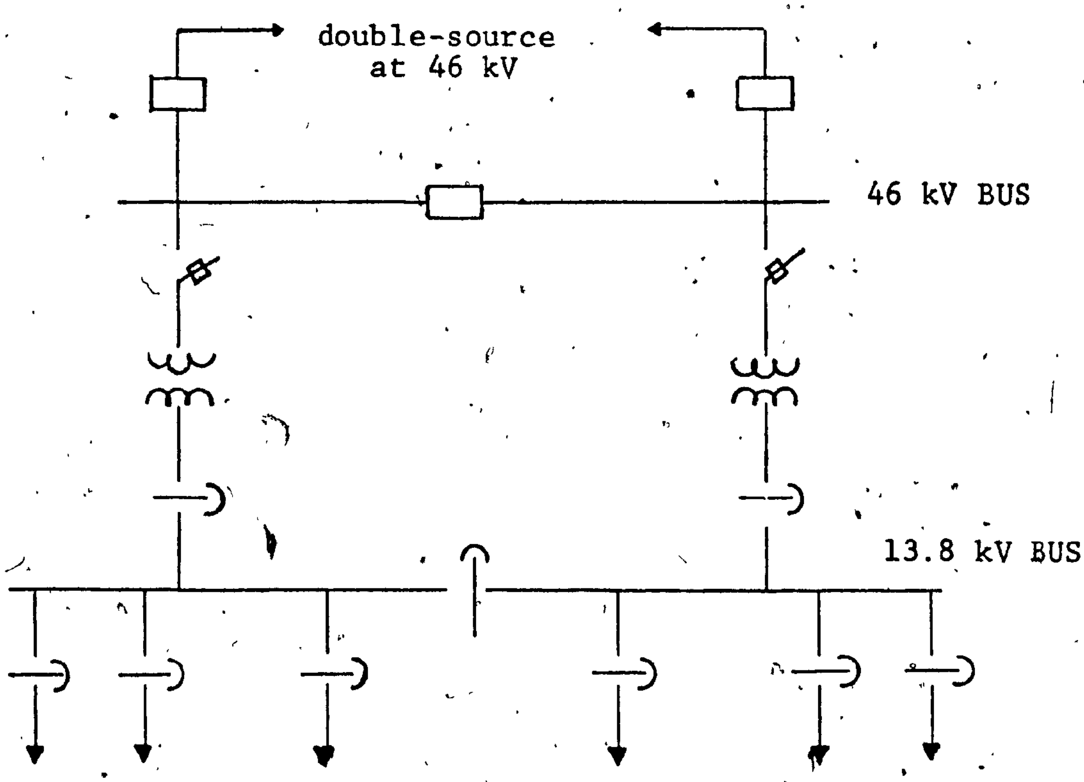

Fig.(3.10) Industrial substation with two sources.

Tables (3.4) and (3.5) show the failure rates and expected outage hours of both substations. It is shown that the failure rate of the first substation is about 10 times as high as that of the second substation; and its expected outage hours about six times as much.

The conclusion of this comparison is that reliability is expensive. Depending on the application, the design engineer makes the decision. If continuous power supply is absolutely necessary, the engineer has to select a high reliability design despite the high cost. If on the other hand continuity is not so crucial an alternative cheaper design may be selected.

Component	Failure Rate λ_c (per year)	Repair Time r_c (h)	Number n_c	Station Failure Rate (per year) $n_c\lambda_c$	Station Expected Outage $n_c\, \lambda_c \cdot r_c$ (hours per year)
46-kV bus	0.002	1.2	1	0.002	0.0024
46-kV disconnect	0.001	1.5	2	0.002	0.003
46-kV transformer	0.004	5*	2	0.008	0.040
13.8-kV bus CB	0.010 !	3.5	1	0.010	0.035
13.8-kV bus enclosed	0.002	1.2	1	0.002	0.0024
13.8-kV feeder breaker (fault)	0.002	3.5	6	0.012	0.042
Station caused 13.8-kV outages (subtotal)				0.036	0.1248
46-kV feeder	0.01 per kilometre year	1.3	15 km	0.15	0.195
Total				0.186	0.3198

* time to disconnect faulted unit.

! rate for faults and all unscheduled outages.

Table (3.4). Failure rates for industrial substation with one source.

Define D to be the determinant of the matrix formed by deleting the nth row and column from $[A]$. This gives:

$$D = \begin{vmatrix} -\sum_{j=2}^{n} \rho_{1,j} & \cdots & \rho_{n-1,1} \\ \vdots & & \vdots \\ \rho_{1,n-1} & \cdots & -\sum_{\substack{j=1 \\ j\neq n-1}}^{n} \rho_{n-1,j} \end{vmatrix} \tag{4.9}$$

Also define $\tilde{D}$ as follows, where $P_i(0)$ is the initial condition of the ith non-absorbing state.

$$\tilde{D} = \begin{vmatrix} -\sum_{j=2}^{n} \rho_{1,j} & \cdots & \rho_{n-1,1} & P_1(0) \\ \vdots & & \vdots & \vdots \\ \rho_{1,n-1} & \cdots & -\sum_{\substack{j=1 \\ j\neq n-1}}^{n} \rho_{n-1,j} & P_{n-1}(0) \\ 1 & \cdots & 1 & 0 \end{vmatrix} \tag{4.10}$$

Then the expected passage time T from some particular state to the absorbing state or between residences in the absorbing state is $E(T)$ where:

$$E(T) = \frac{\tilde{D}}{D} \tag{4.11}$$

Thus for the case of the two lines in parallel system of Fig. (4.1) and (4.2) discussed above with $P_2(0) = 1$ and $P_1(0) = P_3(0) = 0$

$$E(T_3') = \frac{\begin{vmatrix} -2\lambda & \mu & 0 \\ 2\lambda & -(\lambda+\mu) & 1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} -2\lambda & \mu \\ 2\lambda & -(\lambda+\mu) \end{vmatrix}} = \frac{2\lambda+\mu}{2\lambda^2} \tag{4.12}$$

Where $E(T_3')$ is the time between residences in the absorbing state 3.

If the expected residence time in state 3 is desired and the states 1 and 2 are considered as absorbing states, the initial conditions would be $P_1(0) = P_2(0) =$

$P_{12}(0) = 0$ and $P_3(0) = 1$ and the expected residence time,

$$E(T_3'') = \frac{\begin{vmatrix} -2\mu & 1 \\ 1 & 0 \end{vmatrix}}{-2\mu} = \frac{1}{2\mu} \tag{4.13}$$

Which is the down-time of the system. The state-space diagram of this case is shown in Fig.(4.3)

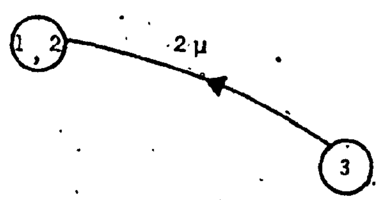

Fig.(4.3). A three-state diagram with two absorbing states.

The MTBF index is a special case of the general form of expected passage and residence times E(T), because, it indicates the average or expected time for the system to go from the "up" state to the "down" state (i.e. failure of the system).

The procedure to calculate the MTBF has as follows. A truncated matrix Q is being formed by deleting the absorbing states from the stochastic transitional matrix.

Then a fundamental matrix N is formed according to the relation $N=(I-Q^{-1})$ where I is the identity matrix and Q^{-1} the inverse of Q. The value of MTBF is then obtained.

Thus going back to the previous example and letting state 3 be the absorbing state (i.e. both lines down), the truncated matrix Q would be:

$$Q = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 1-2\lambda & 2\lambda \\ \mu & 1-(\lambda+\mu) \end{bmatrix} \end{array} \tag{4.14}$$

The transition matrix P being:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 2\mu & 1-2\mu \end{bmatrix} \end{array} \tag{4.15}$$

Then the fundamental matrix N would be:

$$N= (I-Q)^{-1} = \frac{1}{2\lambda^2} \begin{bmatrix} \lambda+\mu & 2\lambda \\ \mu & 2\lambda \end{bmatrix} \tag{4.16}$$

and therefore starting in state 1 the MTBF of the process is $M_{1,3}$.

$$M_{1,3} = \frac{1}{2\lambda^2} \left(\lambda + \mu + 2\lambda \right) = \frac{3\lambda+\mu}{2\lambda^2} \tag{4.17}$$

Note that MTBF is the time before entering the absorbed state 3. Whereas $E(T_3)$ previously found is the time between residences in state 3. This explains the difference in results.

4.3 The Markov processes and their application in transmission systems reliability

The following two examples will illustrate the application of the Markov processes in the reliability of transmission lines. In the first example a simple transmission line is subject to environmental fluctuations. The line is either up or down, and the weather is either normal or stormy. It is assumed that the failure rates, and the weather duration distributions are exponential. The state-space diagram for this case is shown in Fig.(4.4) where:

λ', μ' = stormy weather failure and repair rates.

λ, μ = normal weather failure and repair rates.

$m = 1/S$ where S is the expected duration of a stormy weather period.

$n = 1/N$ where N is the expected duration of a normal weather period.

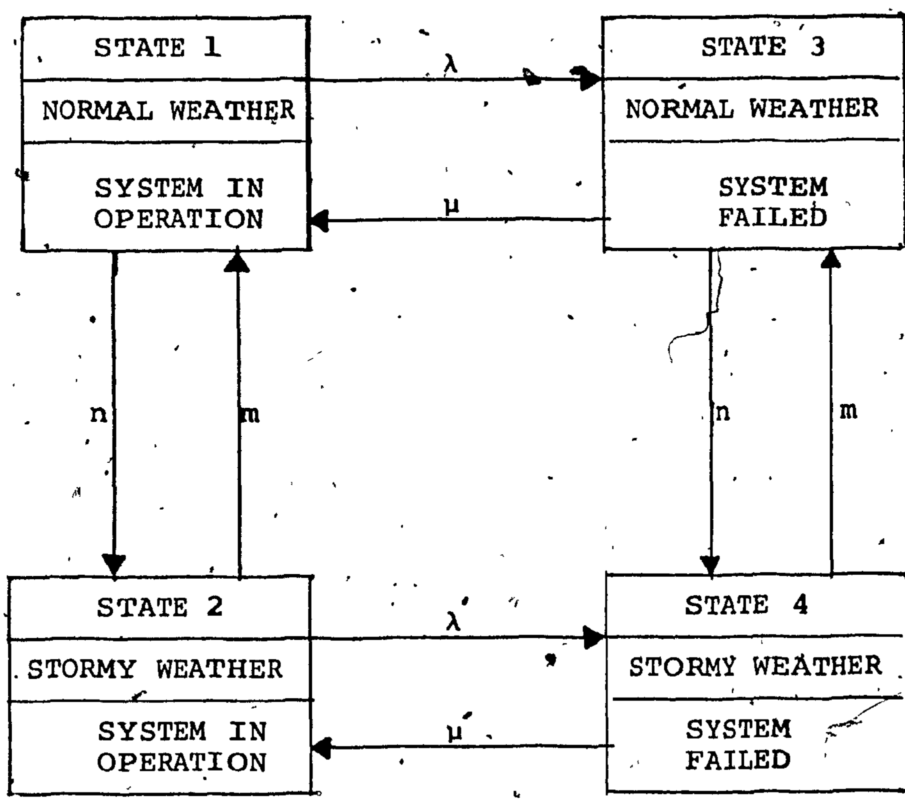

Fig. (4.4) State-space diagram for the single-unit case with a 2-state fluctuating environment.

Proceeding directly from the state-space diagram in Fig. (4.4) the differential matrix form is:

$$\begin{bmatrix} P_1'(t) \\ P_2'(t) \\ P_3'(t) \\ P_4'(t) \end{bmatrix} = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix} \begin{bmatrix} -(\lambda+n) & m & \mu & 0 \\ n & -(m+\lambda') & 0 & \mu' \\ \lambda & 0 & -(\mu+n) & m \\ 0 & \lambda' & n & -(\mu'+m) \end{bmatrix} \qquad (4.18)$$

The stochastic transition matrix P is:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1-(\lambda+n) & n & \lambda & 0 \\ m & 1-(m+\lambda') & 0 & \lambda' \\ \mu & 0 & 1-(\mu+n) & n \\ 0 & \mu' & m & 1-(\mu'+m) \end{bmatrix} \end{array} \qquad (4.19)$$

The truncated Q matrix is formed by deleting states 3 and 4 (system down) which are the absorbing states, or,

$$Q = \begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 1-(\lambda+n) & n \\ m & 1-(m+\lambda') \end{bmatrix}$$

and finally, the fundamental matrix N is given by:

$$N = (I-Q^{-1}) = \frac{1}{\lambda'\lambda + \lambda m + \lambda' n} \begin{bmatrix} m+\lambda' & n \\ m & \lambda+n \end{bmatrix} \qquad (4.20)$$

Starting in state 1 the MTBF ($M_{1,34}$) is:

$$M_{1,34} = \frac{m + \lambda' + n}{\lambda\lambda' + \lambda m + \lambda' n} \tag{4.21}$$

Considering normal weather only, $\lambda'=0$ $m=1$ and $n=0$ therefore $M_{1,34} = 1/\lambda$ as expected for a single unit.

The expected failure rate λ_{av} is by definition $1/M$ thus,

$$\lambda_{av} = \frac{\lambda\lambda' + \lambda m + \lambda' n}{\lambda' + m + n} \tag{4.22}$$

and since $\lambda\lambda' << \lambda m + \lambda' n$

and $\lambda' << m + n$

then,

$$\lambda_{av} \approx \frac{\lambda m}{m + n} + \frac{\lambda' n}{m + n}$$

or,

$$\lambda_{av} \approx \frac{\lambda N}{S + N} + \frac{\lambda' S}{S + N} \tag{4.23}$$

which is the same as in the approximate method which will be discussed later in this chapter.

The second example studies two transmission lines in series exposed to normal and stormy weather. Fig. (4.5) shows the state-space diagram, from which the differential equations in matrix form are:

$$\left| P'(t) \right| = \left| P(t) \right| \left| \begin{array}{c:c} D_1 & m(I) \\ \hdashline m(I) & D_2 \end{array} \right| \tag{4.24}$$

where I is the identity matrix and,

$$\left| D_1 \right| = \left| \begin{array}{cccc} -\lambda_1-\lambda_2-n & \lambda_2 & \lambda_1 & 0 \\ \mu_2 & -\lambda_1-\mu_2-n & 0 & \lambda_1 \\ \mu_1 & 0 & -\mu_1-\lambda_2-n & \lambda_2 \\ 0 & \mu_1 & \mu_2 & -\mu_1-\mu_2-n \end{array} \right| \tag{4.25}$$

$$\left| D_2 \right| = \left| \begin{array}{cccc} -\lambda'_1-\lambda'_2-m & \lambda'_2 & \lambda'_1 & 0 \\ \mu'_2 & -\lambda'_1-\mu'_2-m & 0 & \lambda'_1 \\ \mu'_1 & 0 & -\lambda'_2-\mu'_1-m & \lambda'_2 \\ 0 & \mu' & \mu_2 & -\mu'_1-\mu'_2-m \end{array} \right| \tag{4.26}$$

λ, μ have their usual meaning and the subscripts denote the component number.

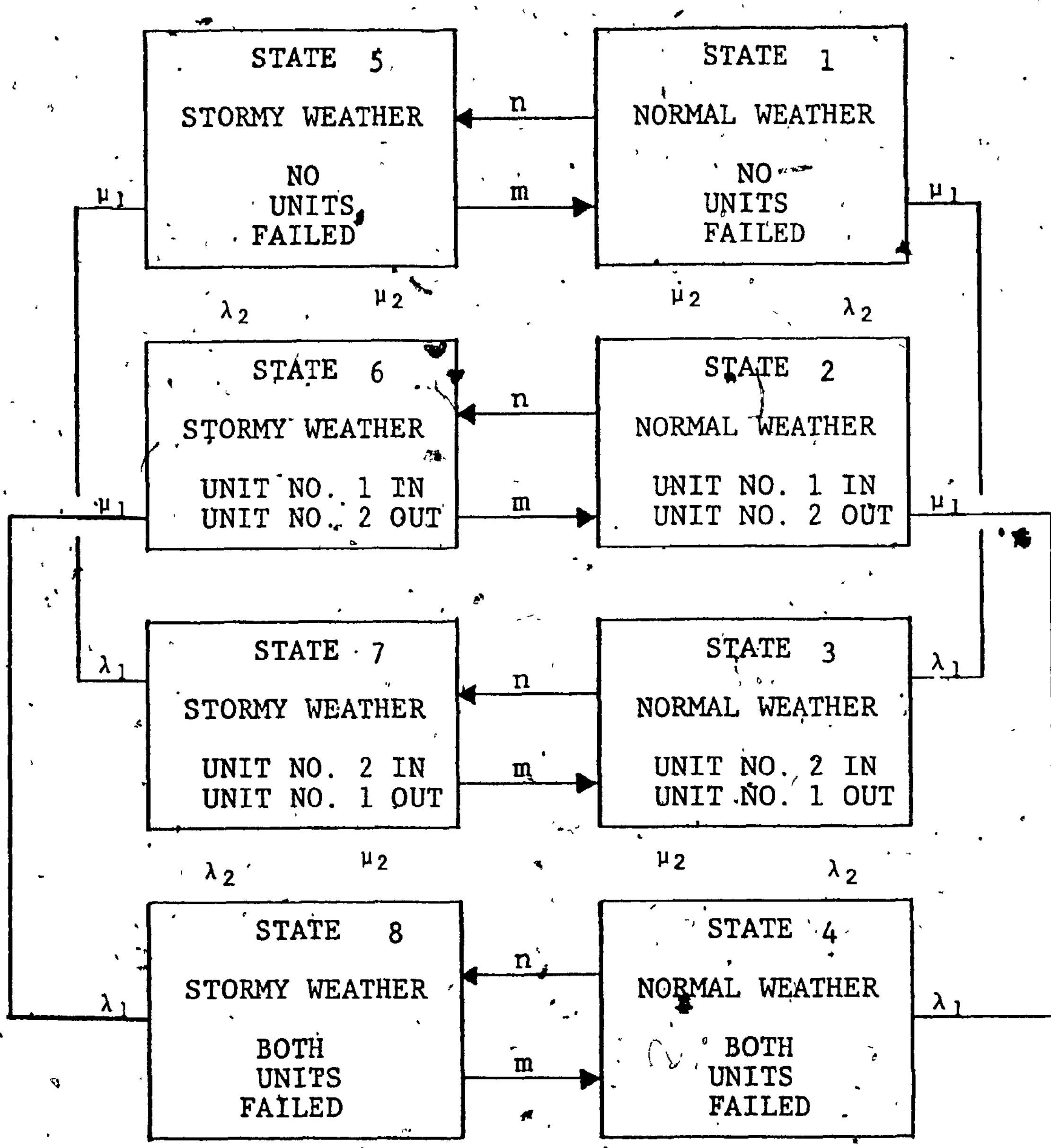

Fig. (4.5) State-space diagram for the 2-unit case with a 2-state fluctuating environment.

States 2,3,4,6,7,8 are absorbing states and the fundamental matrix N is found to be:

$$N = (I-Q)^{-1} = \frac{\begin{vmatrix} \lambda_1'+\lambda_2'+m & m \\ m & \lambda_1+\lambda_2+m \end{vmatrix}}{\lambda_1\lambda_2' + \lambda_1\lambda_1' + \lambda_1 m + \lambda_2\lambda_2' + \lambda_2\lambda_1' + \lambda_2 m + \lambda_2' n + \lambda_1' n} \tag{4.27}$$

This implies that the MTBF starting from state, 1 and letting $\lambda_1=\lambda_2=\lambda$ and $\lambda_1'=\lambda_2'=\lambda'$ is,

$$M_1 = \frac{2\lambda' + m + n}{2(2\lambda\lambda' + \lambda m + \lambda' n)} \tag{4.28}$$

which for $\lambda'=0$, $m=1$ and $n=0$ (i.e. normal weather) gives $M_1 = 1/2\lambda$ as expected for the two unit system. Again the expected failure rate is:

$$\lambda_{av} = \frac{2(2\lambda\lambda' + \lambda m + \lambda' n)}{(2\lambda' + m + n)} \tag{4.29}$$

and neglecting the terms $2\lambda\lambda'$ and $2\lambda'$ as being very small,

$$\lambda_{av} \approx \frac{2\lambda m}{m+n} + \frac{2\lambda' n}{m+n} = 2\frac{\lambda N}{S+N} + \frac{\lambda' S}{S+N} \tag{4.30}$$

which is twice as before, as expected. Note that if the lines were connected in parallel states 4 and 8 would have been the absorbing states. Also note that it would have been extremely laborious to obtain a general expression, for the two units in parallel case, whereas a numerical solution would be relatively easy if a computer is to be used.

Finally if three lines connected in series in a 2-state fluctuating environment are studied, states 2,3,4, 5,6,7,9,10,11,12,13,14,15,16 are absorbing states. The state-space diagram is shown in Fig.(4.6) and the differential equations in matrix form are:

$$\left| P'(t) \right| = \left| P(t) \right| \left| \begin{array}{c|c} D_3 & n(I) \\ \hline m(I) & D_4 \end{array} \right| \tag{4.31}$$

where:

$$\left| D_3 \right| = \begin{vmatrix} A_1 & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 \\ \mu_1 & A_2 & 0 & 0 & \lambda_2 & \lambda_3 & 0 & 0 \\ \mu_2 & 0 & A_3 & 0 & \lambda_1 & 0 & \lambda_3 & 0 \\ \mu_3 & 0 & 0 & A_4 & 0 & \lambda_1 & \lambda_2 & 0 \\ 0 & \mu_2 & \mu_1 & 0 & A_5 & 0 & 0 & \lambda_3 \\ 0 & \mu_3 & 0 & \mu_1 & 0 & A_6 & 0 & \lambda_2 \\ 0 & 0 & \mu_3 & \mu_2 & 0 & 0 & A_7 & \lambda_1 \\ 0 & 0 & 0 & 0 & \mu_3 & \mu_2 & \mu_1 & A_8 \end{vmatrix} \tag{4.32}$$

$$\left| D'_4 \right| = \begin{vmatrix} A'_1 & \lambda'_1 & \lambda'_2 & \lambda'_3 & 0 & 0 & 0 & 0 \\ \mu'_1 & A'_2 & 0 & 0 & \lambda'_2 & \lambda'_3 & 0 & 0 \\ \mu'_2 & 0 & A'_3 & 0 & \lambda'_1 & 0 & \lambda'_3 & 0 \\ \mu'_3 & 0 & 0 & A'_4 & 0 & \lambda'_1 & \lambda'_2 & 0 \\ 0 & \mu'_2 & \mu'_1 & 0 & A'_5 & 0 & 0 & \lambda'_3 \\ 0 & \mu'_3 & 0 & \mu'_1 & 0 & A'_6 & 0 & \lambda'_2 \\ 0 & 0 & \mu'_3 & \mu'_2 & 0 & 0 & A'_7 & \lambda'_1 \\ 0 & 0 & 0 & 0 & \mu'_3 & \mu'_2 & \mu'_1 & A'_8 \end{vmatrix} \qquad (4.33)$$

where:

$$\begin{aligned}
A_1 &= -\lambda_1-\lambda_2-\lambda_3-n & \qquad A'_1 &= -\lambda'_1-\lambda'_2-\lambda'_3-m \\
A_2 &= -\mu_1-\lambda_2-\lambda_3-n & A'_2 &= -\mu'_1-\lambda'_2-\lambda'_3-m \\
A_3 &= -\mu_2-\lambda_1-\lambda_1-n & A'_3 &= -\mu'_2-\lambda'_1-\lambda'_1-m \\
A_4 &= -\mu_3-\lambda_1-\lambda_2-n & A'_4 &= -\mu'_3-\lambda'_1-\lambda'_2-m \\
A_5 &= -\mu_1-\mu_2-\lambda_3-n & A'_5 &= -\mu'_1-\mu'_2-\lambda'_3-m \\
A_6 &= -\mu_1-\mu_3-\lambda_2-n & A'_6 &= -\mu'_1-\mu'_3-\lambda'_2-m \\
A_7 &= -\mu_2-\mu_3-\lambda_1-n & A'_7 &= -\mu'_2-\mu'_3-\lambda'_1-m \\
A_8 &= -\mu_1-\mu_2-\mu_3-n & A'_8 &= -\mu'_1-\mu'_2-\mu'_3-m
\end{aligned}$$

Note that the absorbing states, had the lines been connected in parallel, would have been 16 and 8. The state-space diagram of Fig.(4.6) for the three line system can be applied to the case of the two parallel lines in series with a third line.

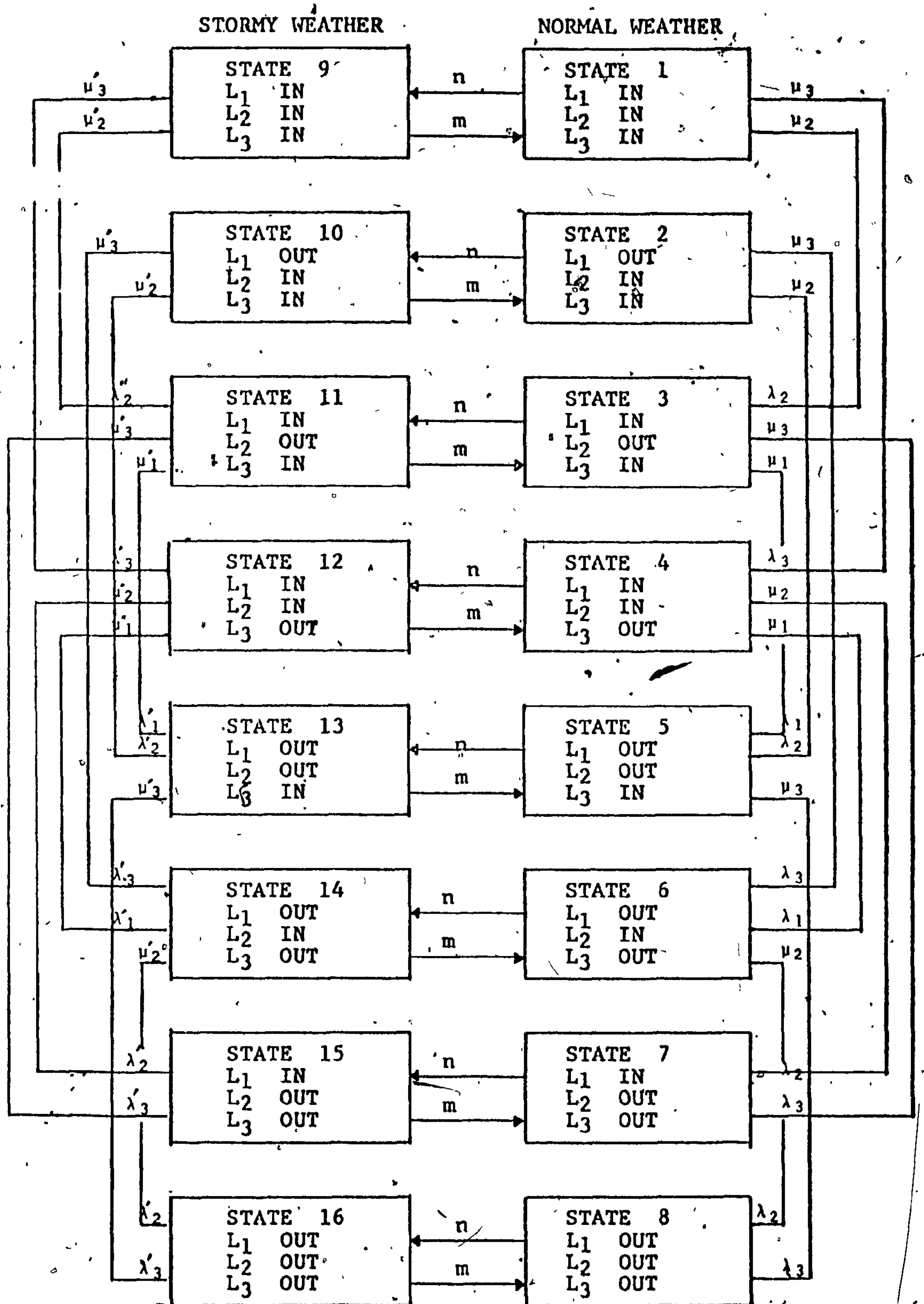

Fig. (4.6) State-space diagram for the 3-unit case with a 2-state fluctuating environment.

The system is shown in Fig.(4.7). For this configuration the failure rates for lines 1 and 2 are 0.5 failure per year. The expected duration of normal, stormy and repair times are 200 hours, 1.5 hours, and 7.5 hours respectively.

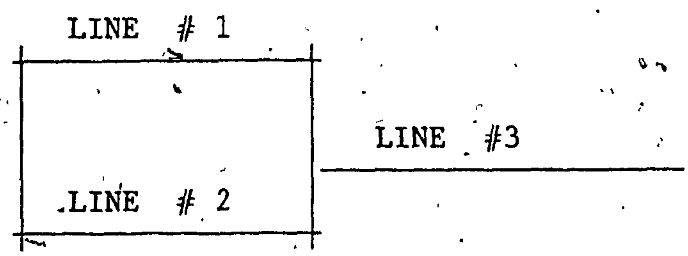

Fig.(4.7). Series-parallel configuration.

The failure rate for the series component varies and the configuration failure rate is calculated with the percentage of storm associated failures held at 50 percent for all components. The variation in configuration failure rate with the failure rate of the series component is shown in Fig.(4.8). It is interesting to note that the series component failure rate almost completely dominates the configuration value even for relatively small series component failure rates.

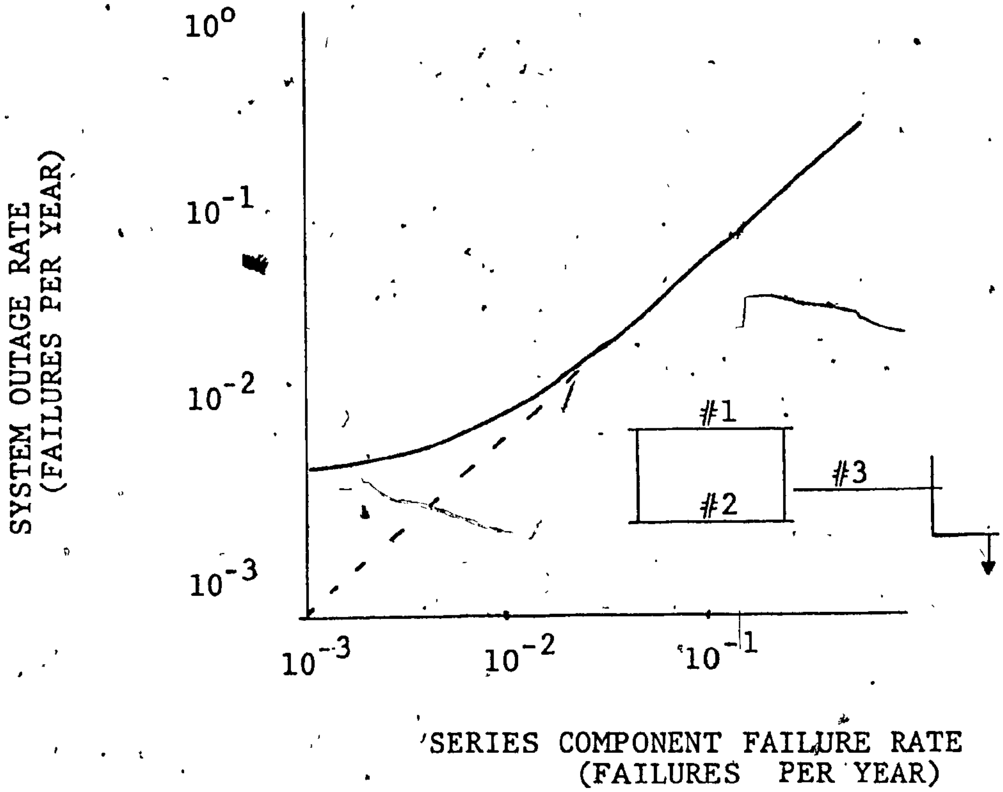

Fig. (4.8). System failure rate for series-parallel configuration showing the dominating effect of the series component on the overall outage rate of the system.

4.4 The Approximate Method

The approximate method is used to calculate the reliability of transmission system from the basic system component parameters, and a characterization of environmental severity variations. Component parameters include the failure and repair rates and weather characterization accounts for normal and severe weather duration periods.

A number of assumptions is made in the process of deriving the mathematical models (thus the name approximate method). These assumptions are:

a) Repair times are negligible compared with times to failure and times between storms.

b) The times to failure, the repair times, the durations of periods of normal and stormy weather and the maintenance down times are exponentially distributed.

c) The duration of severe weather is very short compared with times to failure for components and repair times.

d) In parallel systems, once a line is overloaded, it remains overloaded and down until a failed parallel component is repaired.

4.4.1 Series Systems

In this section a general expression for the measure of reliability in series systems will be presented. A system with n dissimilar components connected in series will be considered. The system has the following parameters:

$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ = Normal weather component failure rates per year of normal weather.

$\lambda'_1, \lambda'_2, \lambda'_3, \ldots, \lambda'_n$ = Stormy weather component failure rate per year of stormy weather.

$\lambda''_1, \lambda''_2, \ldots, \lambda''_n$ = Component maintenance outage per calendar year.

$r_1, r_2, \ldots, r_n$ = Expected repair time for all forced outages in years.

$r''_1, r''_2, \ldots, r''_n$ = expected downtime for maintenance outages in years.

N = expected normal weather duration in years.

S = expected duration of stormy weather in years.

The overall forced outage rate for the ith component is:

$$\lambda_{fi} = \frac{N}{S + N} \lambda_i + \frac{S}{N + S} \lambda'_i \quad \text{(forced outage per calendar year)} \tag{4.34}$$

This is an approximation well justified since $\lambda_i N$ and $\lambda_i S$ are very small compared with unity. The overall outage rate for the series system is:

$$\lambda_{fe} = \sum_{i=1}^{n} \lambda_{fi} \quad \text{(forced outage per calendar year)} \tag{4.35}$$

The maintenace outage rate for the system is:

$$\lambda''_e = \sum_{i=1}^{n} \lambda''_i \quad \text{(maintenance outages per calendar year)} \tag{4.36}$$

Expected values of down time for series systems as results of forced outages and maintenance outages are, respectively:

$$r_{fe} = \frac{\sum_{i=1}^{n} \lambda_{fi}\, r_i}{\lambda_{fe}} \quad \text{(years)} \tag{4.37}$$

and,

$$r''_e = \frac{\sum_{i=1}^{n} \lambda''_i\, r''_i}{\lambda''_e} \quad \text{(years)} \tag{4.38}$$

If the series system is connected in parallel with other components it may be required to replace the series system with an equivalent component e. This equivalent element has:

$$\lambda_e = \sum_{i=1}^{n} \lambda_i \quad \text{(failures per year of normal weather)} \tag{4.39}$$

and,

$$\lambda'_e = \sum_{i=1}^{n} \lambda'_i \quad \text{(failures per year of stormy weather)} \tag{4.40}$$

If a load is attached to the last element and a source to the first element, then the following reliability measures are obtained:

a) Annual outage rate:

$$\lambda_{SL} = \lambda_{fe} + \lambda_e \quad \text{(outages per calendar year)} \tag{4.41}$$

b) Expected outage duration:

$$r_{SL} = \frac{\lambda_{fe}\, r_{fe} + \lambda_e''\, r_e''}{\lambda_{SL}} \quad \text{(year)} \tag{4.42}$$

Which may be converted to hours per year by multiplying it by 8,760.

c) Average total outage time per year:

$$U_{SL} = \frac{r_{SL}}{V_{SL} + \dfrac{1}{\lambda_{SL}}} \doteq \lambda_{SL}\, r_{SL} \quad \text{(years per year)} \tag{4.43}$$

which may also be converted to hours per year if multiplied by 8,760.

d) Probability that a single outage will last longer than t hours:

$$P(\text{outage} > t \text{ hours}) = \sum_{i=1}^{n} \frac{\lambda_{fi}\, e^{-t/(8760\, \tau_i')} + \lambda_i''\, e^{-t/(8760\, \tau_i'')}}{\lambda_{SL}} \tag{4.44}$$

4.4.2 Parallel Systems

The equations to be given here apply to two components in parallel. Thus if the system has three components or more the best way to obtain an overall measure is to treat two components at a time.

The first equation gives the overall outage rate due to normal and stormy weather. Two lines in parallel are considered:

$$\lambda_{SL} = \frac{N}{N + S} \Big(\underbrace{\lambda_1\lambda_2(r_1+r_2) + \frac{S}{N}(\lambda_1'\lambda_2 r_1 + \lambda_2'\lambda_1 r_2)}_{A}$$

$$+ \underbrace{\frac{S}{N}(\lambda_1\lambda_2' r_1 + \lambda_2\lambda_1' r_2) + 2\frac{S^2}{N}\lambda_1'\lambda_2'}_{B} \Big)$$

$$+ \underbrace{\lambda_1''\lambda_2 r_1'' + \lambda_2''\lambda_1 r_2''}_{C} \quad \text{(outages per calendar year)} \tag{4.44}$$

In this expression the term A represents the outage rate λ_e during normal weather. Term B represents the outage rate λ_e during stormy weather and the third term represents the maintenance outages per year.

The expected down time as a result of system outage caused by overlapping component forced outages forced outages is:

$$r_{fe} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{r_1 r_2}{r_1 + r_2}, \text{ (years)} \tag{4.45}$$

and the expected value of system down time when a component forced outage overlaps a component maintenance outage is:

$$r_e'' = \left(\frac{\lambda_1''\lambda_2 r_1''}{\lambda_1''\lambda_2 r_1'' + \lambda_2''\lambda_1 r_2''} \right) \left(\frac{r_2 r_1''}{r_2 + r_1''} \right)$$

$$+ \left(\frac{\lambda_2''\lambda_1 r_2''}{\lambda_1''\lambda_2 r_1'' + \lambda_2''\lambda_1 r_2''} \right) \left(\frac{r_1 r_2''}{r_1 + r_2''} \right) \text{ (years)} \tag{4.46}$$

The first factor of the first term in the above expression represents the fraction of system outages involving component maintenance outages in which component 2 failed while 1 was out for maintenance. The second factor of the first term represents the expected down time given that component 2 fails while 1 is out for maintenance. The second term of the expression accounts in like manner for the situation where a forced outage of component 1 overlaps a maintenance outage of component 2.

The probability that a single outage will last longer than t hours is given by the following expression:

$$P(\text{outage} > t \text{ hours}) = \frac{1}{\lambda_{SL}} \Big((\lambda_{SL} - \lambda''_e)\, e^{-t/(8760\, \tau\, f_e)}$$

$$+ \lambda''_1 \lambda_2 r''_1 \exp\left(-t/\left(8760 \frac{r_2\, r''_1}{r_2 + r''_1}\right)\right)$$

$$+ \lambda''_2 \lambda_1 r''_2 \exp\left(-t/\left(8760 \frac{r''_2\, r_1}{r_1 + r''_2}\right)\right) \Big) \qquad (4.47)$$

This concludes the reliability measures of parallel system under normal and severe weather. The forced and scheduled maintenance have been considered.

The following section will examine another mode of failure, namely the outage due to overload in parallel systems.

4.4.2.1 Outages due to Overload in Parallel Systems

Consider a system composed of elements 1 and 2 in parallel. The overload outage rates for normal and stormy weather are"

$$\lambda_{oe} = \lambda_1 P_2 + \lambda_2 P_1 \qquad \text{(overload outages per year of normal weather)} \tag{4.48}$$

and,

$$\lambda'_{oe} = \lambda'_1 P_2 + \lambda'_2 P_1 \qquad \text{(overload outages per year of stormy weather)} \tag{4.49}$$

where,

P_i = probability that component i will not be able to carry contingency load.

The system overall outage rate due to overload λ_{ofe} is given as:

$$\lambda_{ofe} = \frac{N}{N + S} \lambda_{oe} + \frac{S}{N + S} \lambda'_{oe} \tag{4.50}$$

(overload outages per calendar year)

The probabilities P_1 and P_2 are given as:

$$P_1 = 1 - \int_0^{\infty} Q_1(X)\, dM_2(X) \tag{4.51}$$

and,

$$P_2 = 1 - \int_0^\infty Q_2(X)\, dM_1(X) \tag{4.52}$$

where,

$Q_i(X)$ = probability the component i can successfully carry contingency load for a time X.

$M_i(X)$ = probability that repair time of component i is completed in time X.

= $1 - e^{-X/r_i}$, for exponentially distributed repair times.

A practical $Q_i(X)$ would be equal to $Q_i(24)$ for all X greater than 24 hours and therefore for the case of two components in parallel,

$$P_1 = 1 - Q_1(24)e^{-24/(8760r_2)} - (1-e^{-1/(8760r_2)})$$

$$\times \sum_{j=1}^{24} Q_1(j-1/2)e^{-(j-1)/(8760r_2)} \tag{4.53}$$

and,

$$P_2 = 1 - Q_2(24)e^{-24/(8760r_1)} - (1-e^{-1/(8760r_1)})$$

$$\times \sum_{j=1}^{24} Q_2(j-1/2)e^{-(j-1)/(8760r_1)} \tag{4.54}$$

The values of Q(X) can be obtained from graphs. Fig. (4.9) shows such a graph for time ranging from 0 to 36 hours.[20]

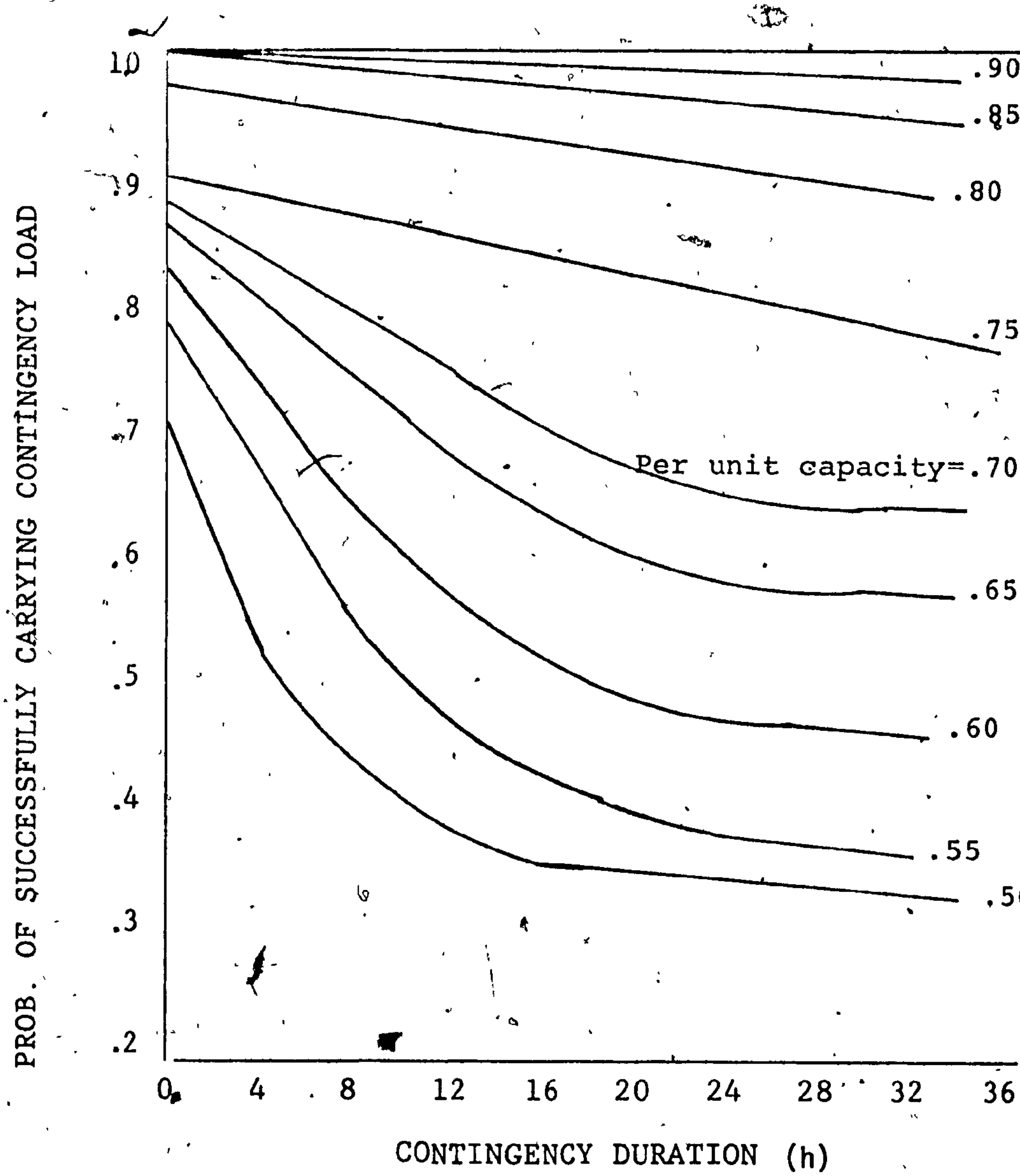

Fig. (4.9) Typical set of Q curves

Summarizing, the system failure due to all types of outages (overlapping forced outages, maintenance associated outages, and overload outages) can be obtained by adding the values found above. Thus, defining λ_{TSL} as parallel system failure rate as a result of all outages,

$$\lambda_{TSL} = \lambda_{SL} + \lambda_{ofe} \qquad \text{(total outages per calendar year)} \tag{4.55}$$

The down time for the two component system is then,

$$r_{TSL} = \frac{\lambda_{fe}}{\lambda_{TSL}} r_{fe} + \frac{\lambda''_{e}}{\lambda_{TSL}} r''_{e} + \frac{\lambda_{f1} P_2}{\lambda_{TSL}} r_1 + \frac{\lambda_{f2} P_1}{\lambda_{TSL}} r_2 \qquad \text{(years)} \tag{4.56}$$

and finally considering all types of outages, the probability that a single system outage in a 2-component parallel system will last longer than 2 hours is,

$$\begin{aligned} P(\text{outage} > t \text{ hours}) = \frac{1}{\lambda_{TSL}} \Big(& \lambda_{fe} \exp(-t/(8760\, r_{fe})) \\ & + \lambda''_1 \lambda_2 r''_1 \exp\Big(-t/\Big(8760 \frac{r_2 \cdot r''_1}{r_2 + r''_1}\Big)\Big) + \lambda_{f1} P_2 \exp(-t/(8760 r_1)) \\ & + \lambda''_2 \lambda_1 r''_2 \exp\Big(-t/\Big(8760 \frac{r_1\, r''_2}{r_1 + r''_2}\Big)\Big) + \lambda_{f2} P_1 \exp(-t/(8760\, r_2))\Big) \end{aligned} \tag{4.57}$$

To illustrate the application of the approximate method, consider the following example.

A subtransmission system is connected as shown in Fig.(4.10).

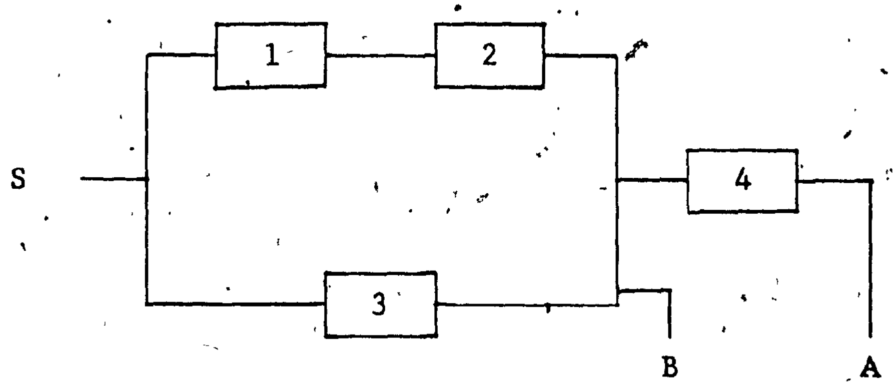

Fig.(4.10) A sample system

The components have the following parameters:

$\lambda_1=\lambda_2=\lambda_3=\lambda_4=$ 0.3 failures per year of normal weather.

$\lambda'_1=\lambda'_2=\lambda'_3=\lambda'_4=$ 15 failures per year of stormy weather.

$\lambda''_1=\lambda''_2=\lambda''_3=$ 2 maintenance outages per year.

$r_1=r_2=r_3=r_4=$ 10^{-3} years.

$r''_1=r''_2=r''_3=r''_4=$ 10^{-3} years.

$\lambda_4'' = 0$

$S = 2.28 \times 10^{-4}$ years = 2 hours.

$N = 2.28 \times 10^{-2}$ years = 200 hours.

The capacities of components 1, 2, 3, are 90 per cent of the peak contingency load that they be required to carry.

It is required to calculate the total outage rate, the expected repair time and the probability that any outage will be longer than 24 hours.

The first step is to combine all series elements into an equivalent component. In this case, components 1, and 2 will be combined to form an equivalent component el. The equivalent failure rates for el would be as follows:

$$\lambda_{el} = \sum_{i=1}^{2} \lambda_i = 0.3 + 0.3 = 0.6$$

$$\lambda'_{el} = \sum_{i=1}^{2} \lambda'_i = 15 + 15 = 30 \quad \text{(failures per year of stormy weather)}$$

$$\lambda''_{el} = \sum_{i=1}^{2} \lambda''_i = 2 + 2 = 4 \quad \text{(maintenance outage per year)}$$

The overall outage rate for el is:

$$\lambda_{fel} = \lambda_{f1} + \lambda_{f2}$$

$$= \frac{N}{S+N}\lambda_1 + \frac{S}{N+S}\lambda_1' + \frac{N}{N+S}\lambda_2 + \frac{S}{N+S}\lambda_2'$$

$= 0.89$ forced outages per year.

Then the expected values for repair times following forced and maintenance outages for components are:

$$r_{fel} = \frac{\sum_{i=1}^{2} \lambda_{fi}\, r_1}{\lambda_{fel}} = 10^{-3} \text{ years}$$

$$r_{el} = \frac{\sum_{i=1}^{2} \lambda_i'' \, r_i''}{\lambda_{el}''} = 10^{-3} \text{ years}$$

The next step is to make all possible parallel reductions. In this step component 3 and equivalent component el will be combined to form equivalent component e2.

The forced and maintenance outage rates would then be:

$$\lambda_{e_2} = \lambda_3 \lambda_{e1} (r_3 + r_{fe1}) + \frac{S}{N} (\lambda_3' \lambda_{e1} r_{fe1} + \lambda_{e1}' \lambda_3 r_{fe1})$$

$= 5.4 \times 10^{-4}$ forced outages per year.

$$\lambda_{e_2}' = \frac{N}{S} \left(\frac{S}{N} (\lambda_3 \lambda_{e1}' r_3 + \lambda_{e1} \lambda_3' r_{fe1}) + 2 \frac{S^2}{N} \lambda_3' \lambda_{e1}' \right)$$

$= 1.902 \times 10^{-2}$ failures per year of stormy weather.

$$\lambda_{e_2}'' = \lambda_3'' \lambda_{e1} r_3'' + \lambda_{e1}'' \lambda_3 r_{e1}''$$

$= 2.4 \times 10^{-3}$ maintenance outages per year.

Then the overall forced outage rate for e_2 would be:

$$\lambda_{fe_2} = \frac{N}{N + S} \lambda e_2 + \frac{S}{N + S} \lambda e_2'$$

$= 7.829 \times 10^{-4}$ forced outages per year.

The expected values of repair times following forced and maintenance associated outages are:

$$r_{fe_2} = \frac{r_3\ r_{fel}}{r_3 + r_{fel}} = 5 \times 10^{-4} \text{ years}$$

and from eq. (4.38),

$$r_{e_2} = 2.5 \times 10^{-4} \text{ years}$$

The probability that components 3 and el cannot successfully carry contingency loads are:

$$P_{el} = 1 - \sum_{i=1}^{24} Q(X_j)\ \left(e^{-(j-1)/8760\ r_3} - e^{-j/(8760\ r_3)} \right)$$

$$- Q(24)\ e^{-24/(8760\ r_3)}$$

$$= 0.06495$$

and,

$$P_3 = 1 - \sum_{j=1}^{24} Q(X_j)\ \left(e^{-(j-1)/8760\ r_{el}} - e^{-j/8760\ r_{el}} \right)$$

$$- Q(24)\ e^{-24/8760\ r_{el}} = 0.06495$$

Then the overload outage rate of component e_2 during normal and stormy weather is:

$$\lambda_{ofe_2} = \frac{N}{N+S} (\lambda_{e1} P_3 + \lambda_3 P_{e1}) + \frac{S}{N+S} (\lambda_{e1} P_3 + \lambda_3 P_{e1})$$

$= 1.0269$ overload outage per year

The total outage rate for equivalent component e_2 due to all modes of failure is:

$$\lambda_{Te_2} = \lambda_{fe_2} + \lambda_{e_2} + \lambda_{ofe_2} = 1.03 \text{ outages}$$

The expected value of repair for component e_2 considering all types of outages is:

$$r_{Te_2} = \frac{\lambda_{fe_2}}{\lambda_{Te_2}} r_{fe_2} + \frac{\lambda_{e_2}}{\lambda_{Te_2}} r_{e_2} + \frac{\lambda_{f3} P_{e1}}{\lambda_{Te_2}} r_3 + \frac{\lambda_{fe1} P_3}{\lambda_{Te_2}} r_{el}$$

$= 28.45$ hours

Thus a customer at point B will experience an average total down time per year,

$$U_B = \frac{r_{Te_2}}{r_{Te_2} + \frac{1}{\lambda_{Te_2}}} = 0.09 \text{ hours per year}$$

and the probability that any outage will be longer than 24 hours is:

$$P(B \text{ out} > 24 \text{ hours}) = e^{-24/r_{Te_2}} = 0.9156$$

Similarly a customer at point A would experience a total outage rate,

$$\lambda_{Te_3} = \lambda_{fe_3} + \lambda_{e_3} = \lambda_{fe_2} + \lambda_{f4} + \lambda_{e_2} + \lambda_4 = 0.4462$$

and an expected value of repair,

$$r_{Te_3} = \frac{\lambda_{fe_3} r_{fe_3} + \lambda_{e_2} r_{e_3}}{\lambda_{Te_3}} = 9.4 \text{ hours}$$

and an average down time,

$$U_A = \frac{r_{Te_3}}{r_{Te_3} + \frac{1}{\lambda_{Te_3}}} = 8.07 \text{ hours per year}$$

The probability that any outage will be longer than 24 hours is:

$$P(\text{A out} > 24 \text{ Hours}) = e^{-24/8.07} = 0.051$$

Finally assuming that there are 3 customers at point B and 4 customers at point A, the following reliability indices are obtained:

a) Average number of interruptions per customer served per year

$$\overline{\lambda} = \frac{\text{No. of Customers at } A(\lambda_{Te_3}) + \text{No. of Customers at } B(\lambda_{Te_2})}{\text{total number of customers}}$$

$= 0.69$ interruptions per year

b) Average customer restoration times

$$\overline{r} = \frac{\text{No. of Customers at } A(r_{Te_3}) + \text{No. of Customers at } B(r_{Te_2})}{\text{total number of customers}}$$

$= 12.7$ hours

c) Average total interruption times per customer served per year

$$\overline{U} = \frac{\text{No. of Customers at } A(U_A) + \text{No. of Customers at } B(U_B)}{\text{total number of customers}}$$

$= 1.44$ hours

d) Maximum expected number of interruptions experienced by any one customer per year

$$\lambda_{max} = \lambda_{Te_3} = 1.0 \text{ outage per year}$$

e) Maximum expected restoration time period by any one customer

$$r_{max} = r_{Te_3} = 24.8 \text{ hours}$$

4.5 Comparison of the two methods

It is interesting to compare the results obtained for expected failure rate and average annual outage duration by the Markov approach with those obtained by the approximate method.

The effect of changes in expected repair time is shown in fig.(4.11) for the two and three parallel line configurations. The effect of varying the expected duration of normal and stormy weather periods is shown in fig.(4.12). The effect of parallel component failure rate variation is shown in fig.(4.13). The approximate method responds quite differently to changes in the system parameters and therefore no overall statement can be made regarding the actual differences except in specific cases. This is clearly shown in figures (4.11) through (4.13).

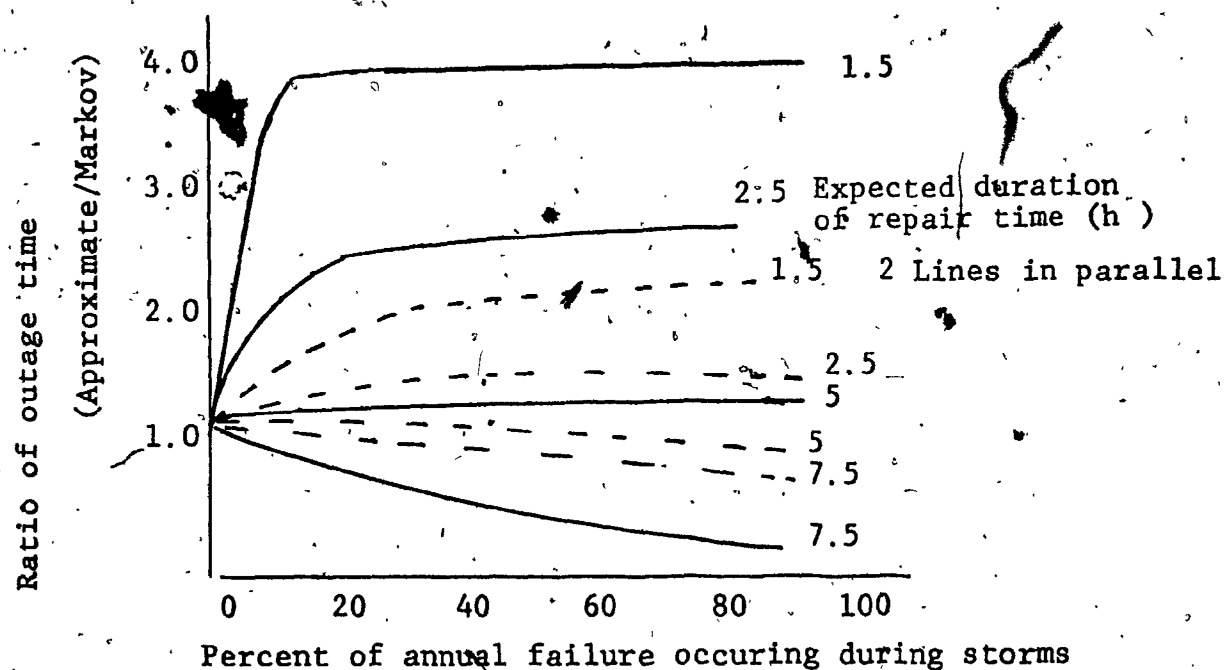

Fig.(4.11). Comparison of the approximate and Markov methods for N=200 hours, S=1.5 hours, and λ_{AV}(component annual failure rate) = 0.5 failure per year.

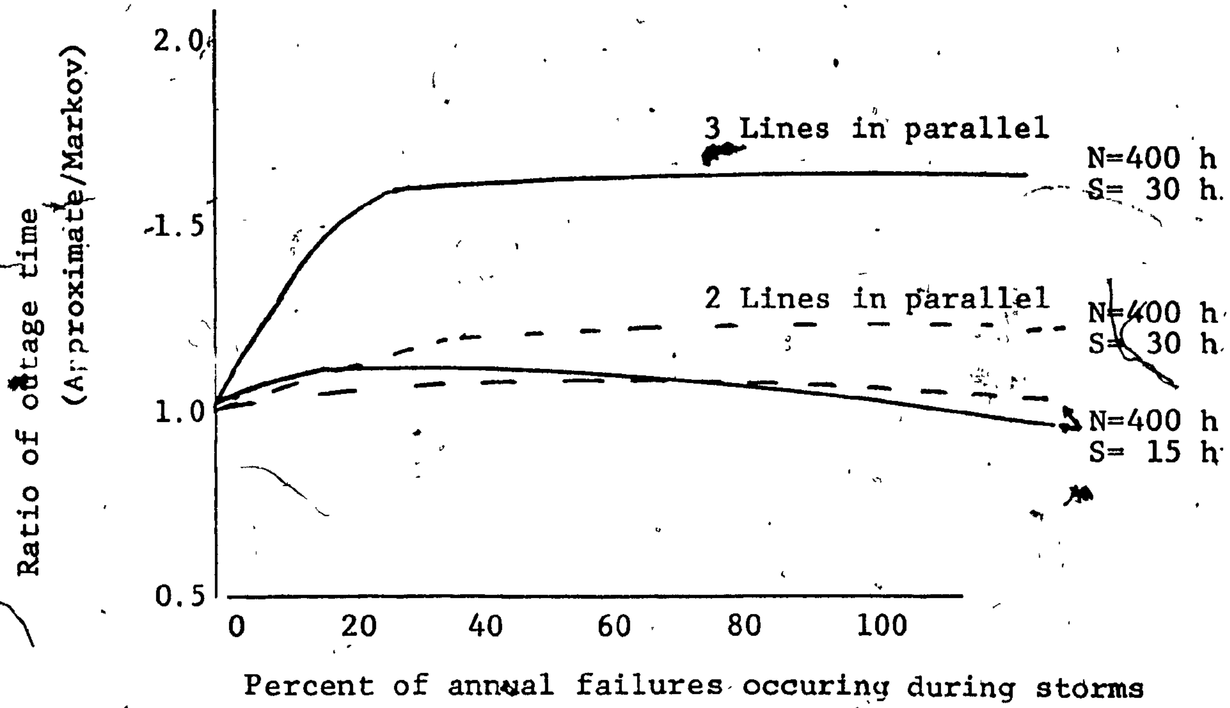

Fig. (4.12). Comparison of the approximate and Markov methods for R=7.5 hours and λ_{AV} (component annual failure rate) = 0.5 failure per year.

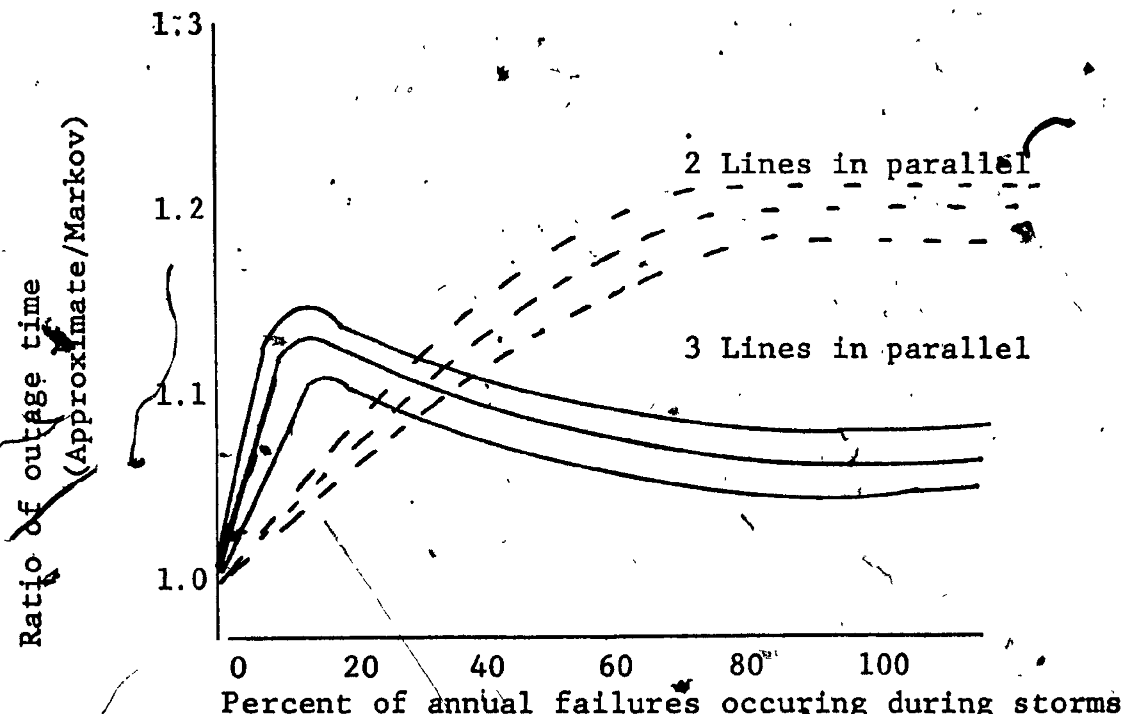

Fig. (4.13). Comparison of the approximate and Markov methods for N=200 hours, S=1.5 hours, and R=7.5 hours.

CHAPTER V

THE RELIABILITY OF POWER GENERATION SYSTEMS

5.1 Introduction

The scope of this chapter is to study the reliability of power generating systems. First, the reliability of the generating capacity is studied. Expressions for the availability failure rate, and frequency of occurence are established for the exact and cumulative states. A method to develop a load model follows along with expressions for availability, failure rate and frequency. Finally, the concept of margin rates is introduced and studied.

Based on the results obtained in the study of the generating capacity and the load model, the expressions for availability, rate of departure and frequency of occurence are defined. This includes the exact state, the equal capacity state and the cumulative state.

5.2 Power Generation Reliability Techniques

In this study of generation requirements, three reliability techniques are found in widespread use.[21,22]

The loss-of-load method is basically a calculation of the probability of the failure to be able to serve the expected peak load over a specified time period. This technique includes the nature of the expected load and usually characterizes each individual generation unit by a maximum capability and a long-run probability of being in service, i.e. its availability.

The second technique is the frequency and duration technique. This method utilizes more data about each generation unit. The method allows the computation of the long term probability of the generation system suffering an outage state of exactly a given amount and the expected frequency with which this state will reoccur.

The third method is the recursive technique method. This method leads to reliability measures of the availability, frequency of occurence and the mean duration of reserve states. These are cumulative states in that they specify system reserve conditions of a given magnitude or less. The loss-of-load method and the recursive method give the same results, whereas the results obtained by the frequency and duration technique are slightly different.

This chapter will examine the recursive technique only. First, a generation model will be studied, followed by a load model analysis. The results obtained, will be used to establish margin availability tables and transfer rates.

5.3 Frequency and Duration by Recursive Techniques

The basic concept of this technique is that the ability of a generator to provide power, is equal to its instantaneous capacity. This is a value changing in time and dependent upon the state of the auxiliary equipment associated with the generator and the environment about the plant. The above statement implies that the output of the generator occupies discrete capacity levels ranging from zero value up to machine rated capacity.

The transition from one capacity state to another is assumed to take place instantaneously and it can occur any time. The amount of time that the capacity remains at a particular value is the time in residence for this state. This defines the availability as the mean-time-in-residence divided by the mean cycle of time for this particular state to occur or reoccur. Fig.(5.1) shows this phenomenon. The upper part represents the average history of unit capability. The space between the shaded blocks represents transition into the states which may be one or many. The lower part is a two-state transition diagram for a repairable machine.

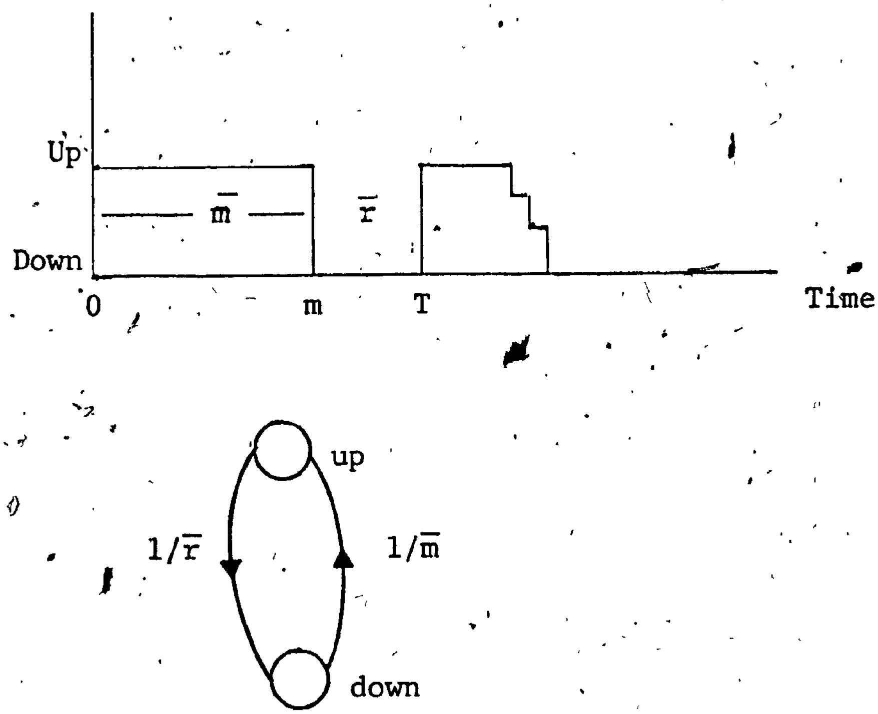

Fig.(5.1). Two state generation model.

The two states are the "up" and "down" states. Assuming that the failure and repair times are constant, this machine state description is a Markov process. The concepts and mathematics of Markov processes were given in chapter IV along with some examples. More examples will be presented later in this chapter.

The next step here is to obtain expressions for frequency of occurence, cycle time and availability for different generation schemes. To achieve this, consider two generators connected in parallel. Assume that both machines are repairable and they have the following parameters:

$T = 1/f$, cycle time (days),

f = frequency (cycle per unit time),

$m = 1/\lambda$, mean uptime (days),

$r = 1/\mu$, mean repair time (days),

λ = failure rate (failures per unit time),

μ = repair rate (repairs per unit time),

$A = m/(m + r) = m/T$, availability (steady state),

and $\bar{A} = 1 - A = r/T$, unavailability (steady state).

Consequently, the following relation can be established:

$$\lambda = 1/AT \tag{5.1}$$

$$\mu = 1/\bar{A}T \tag{5.2}$$

and $$f = A\lambda = \bar{A}\mu \tag{5.3}$$

It is required to find the exact outage states and the cumulative outage states. (Encoutering an outage of exactly 10 MW in one day is an exact outage state whereas encountering an outage of 10 MW or more is a cumulative outage state).

Fig.(5.2) shows a four-state transition diagram for two repairable generators in parallel and fig.(5.3) shows the relationship of exact and cumulative state descriptions.

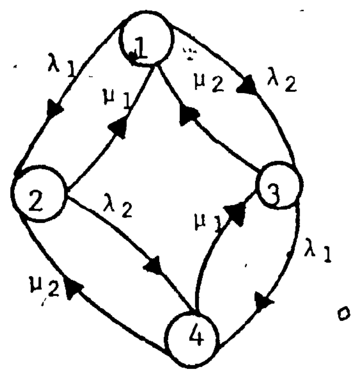

Fig.(5.2) Four-state transition diagram for two generators in parallel.

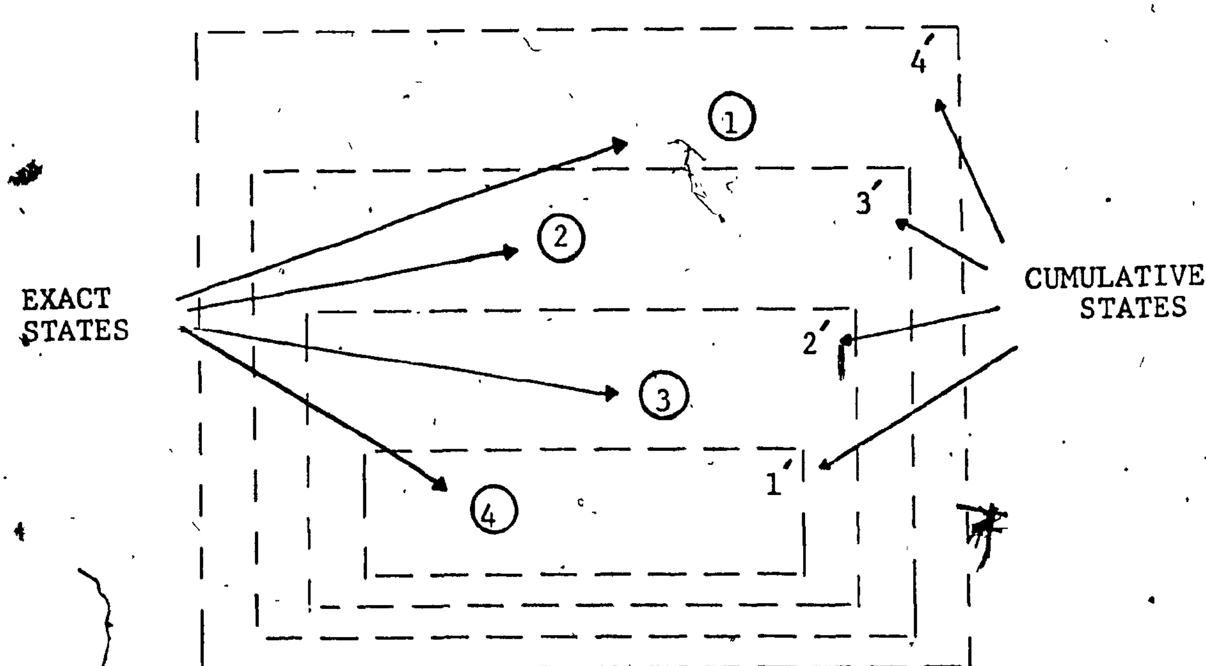

Fig.(5.3) Relationship of exact and cumulative state descriptions for the two parallel generators.

The cumulative states are denoted by primes. Note that, the frequency of encountering state 1 is the same as that of encountering state 4', or:

$$f_{1'} = A_4 (\mu_1 + \mu_2) = f_4 \qquad (5.4)$$

and the frequency of encountering the other cumulative states is given by the general formula:

$$f_n = f_{(n-1)} - A_k \lambda_{-k} + A_k \lambda_{+k} \qquad (5.5)$$

where A_k is the availability of the exact capacity state k.

$\lambda_{+k} = \lambda_{up}$ = rate of transition out of a given capacity state k to one in which more capacity is available

and

$\lambda_{-k} = \lambda_{down}$ = rate of transition out of a given capacity state k to one in which less capacity is available.

Thus from eq. (5.5),

$$f_{2'} = f_{1'} - A_3 \lambda_1 + A_3 \mu_2 \qquad (5.6)$$

The cycle time of encountering an exact state is the reciprocal of the availability of that exact state multiplied by the rate of departure of that state. The cycle time is:

$$T = \frac{1}{A_2\ (\lambda_2 + \mu_1)} \tag{5.7}$$

so if, for example, the capacity of the first generator is 20 MW and the capacity of the second generator is 30 MW with A = 0.98 and r= 2.040816 days, the availabilities and mean times between encountering the states can be calculated. Tables (5.1) and (5.2) show the results.

Unit	Capacity (MW)	Availability	r (days)	μ (per day)	λ (per day)
1	20	0.9800	2.040816	0.49	0.01
2	30	0.9800	2.040816	0.49	0.01

Table (5.1). Reliability parameters for the two generators.

State Number	Capacity Available (MW)	A (per unit)	Rate of Departure (per day)	Cycle Time (days)
1	50	0.9604	$\lambda_1+\lambda_2=0.02$	52.0616
2	30	0.0196	$\mu_1+\lambda_2=0.50$	102.0408
3	20	0.0196	$\lambda_1+\mu_2=0.50$	102.0408
4	0	0.0004	$\mu_1+\mu_2=0.98$	2551.02

Table (5.2). Rate of departure and cycle time for the two generators.

For the sake of completeness, the availability of a cumulative state n may also be found from the relationship:

$$A_n = A_{n-1} + A_k \tag{5.8}$$

where, k is an exact capacity state being appended to the cumulative state n-1 to arrive at n. Table (5.3) and (5.4) show the values obtained using this technique.

Exact Capacity States			Departure Rates	
State Number	Capacity (MW)	Availability	λup (per day)	λdown (per day)
1	50	0.9604	0	0.02
2	30	0.0196	0.490	0.01
3	20	0.0196	0.490	0.01
4	0	0.0004	0.980	0

Table (5.3). Exact capacity states for the two machines.

Cumulative Capacity States			
State Number	Capacity (MW)	Availability	Cycle Time (days)
4	50	1.0000	
3	30	0.0396	52.0616
2	20	0.0200	102.0408
1	0	0.0004	2551.02

Table (5.4). Cumulative capacity states for the two machines.

In the above example, the capacities of the machines were different and the approach used is general.

Identical capacity states is a special case. It is assumed that all machines have the same capacity level and the merged capacity is equal to that capacity level. Also the availability and frequency of the merged state are equal to the sum of the individual state availabilities and frequencies respectively. Thus, if two machines are studied with the two states designated as j and i and k designates the merged state, the following relations are obtained:

a) Capacity,

$$C_k = C_i = C_j \tag{5.9}$$

b) Availability,

$$A_k = A_i + A_j \tag{5.10}$$

c) Frequency,

$$f_k = f_i + f_j \tag{5.11}$$

And the total rates of departure to greater and lesser capacity states may be fround from:

$$A_k \lambda_{up,k} = A_i \lambda_{up,k} + A_j \lambda_{up,j} \tag{5.12}$$

and,

$$A_k \lambda_{down,k} = A_i \lambda_{down,i} + A_j \lambda_{down,j} \quad (5.13)$$

These relationships permit the analysis of non-redundant systems connected in parallel. Namely the exact state availability and frequency of occurence can be calculated. The cumulative capacity quantities can be generated recursively.

The following two examples show the practical application of the above. The first example examines five generators with characteristics as shown in table (5.5).

	Capacity (MW)	Mean Repair Time (days)	Availability (per unit)
	20	2.040816	0.980
	30	2.040816	0.980
	40	5.1082	0.975
	50	5.1543	0.970
	60	5.1543	0.970
Total	200		

Table (5.5). Machine characteristics of the five machines example.

The frequency and the cycle time are shown in fig. (5.4). The cycle time in this case is the mean time to recurrence of an outage of a given magnitude or more.

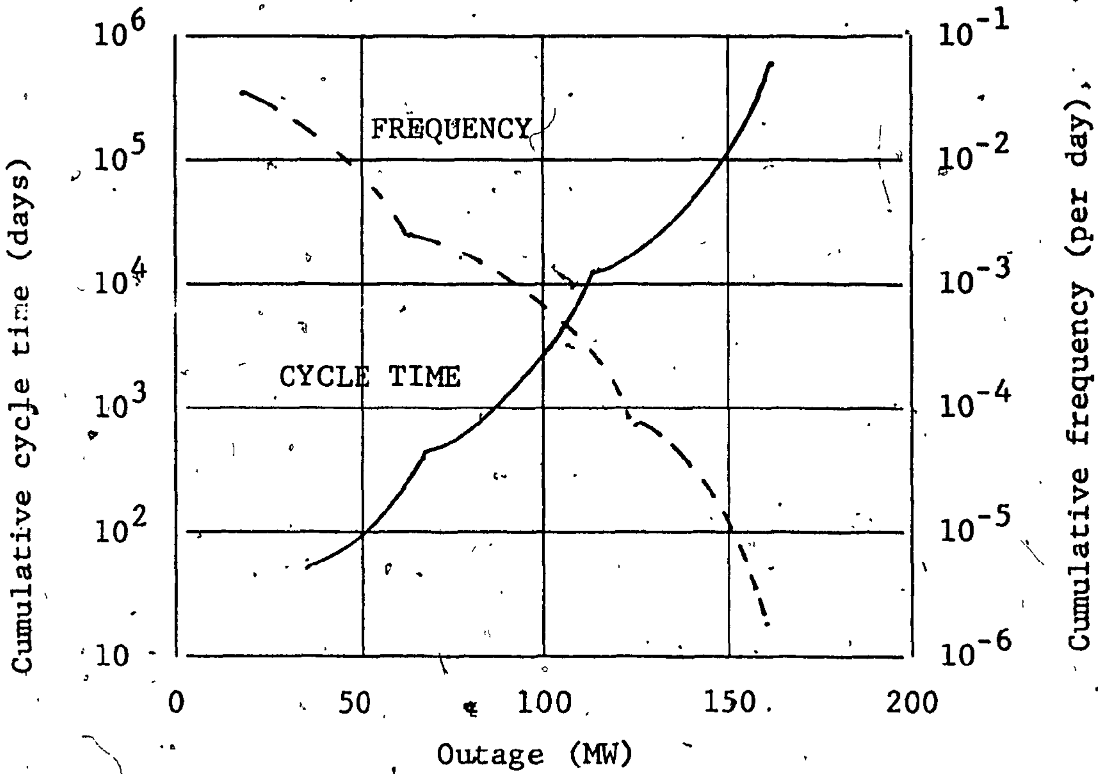

Fig.(5.4). Cumulative state data for five machine system.

The frequency is the reciprocal of the recurrence time. Table (5.6) shows the exact and the cumulative outage states. The availability and cycle time are presented.

The second example examines a system that consists of 20 generators of identical capacity states. The mean repair time is assumed to be 5 days and the forced outage rate is 0.02. Using equation (5.3) along with the availabilities, the transfer rates and the recursive equations (5.9) and (5.13), table (5.7) was constructed.

Capacity Outage (MW)	Exact Outage States: Availability	Exact Outage States: Cycle Time (days)	Cumulative Outage States: Availability	Cumulative Outage States: Cycle Time (days)
0	0.88105	30.659	1.00000	-
20	0.01798	107.57	0.11895	30.659
30	0.01798	107.57	0.10097	41.166
40	0.02259	194.35	0.08299	62.627
50	0.02762	153.90	0.06040	81.514
60	0.02771	154.84	0.03278	133.81
70	0.10171×10^{-2}	1392.0	0.50726×10^{-2}	366.87
80	0.11122×10^{-2}	1275.3	0.40554×10^{-2}	487.56
90	0.12642×10^{-2}	1441.4	0.29432×10^{-2}	761.24
100	0.71004×10^{-3}	3290.0	0.16790×10^{-2}	1470.9
110	0.86836×10^{-3}	2671.5	0.96897×10^{-3}	2424.4
120	0.28515×10^{-4}	3.9145×10^{4}	0.10061×10^{-3}	1.2339×10^{4}
130	0.31458×10^{-4}	3.5546×10^{4}	0.72090×10^{-4}	1.7728×10^{4}
140	0.17490×10^{-4}	6.3452×10^{4}	0.40632×10^{-4}	3.4188×10^{4}
150	0.21900×10^{-4}	7.4362×10^{4}	0.23142×10^{-4}	7.1375×10^{4}

Table (5.6). Cycle times and availabilities for exact and cumulative outage states for the five-machine system.

Number in Service	Availability	Cumulative State Results Cycle Time (days)	Mean Time-to-Failure-MTTF (days)	Mean Time-to-Repair-MTTR (days)
20	1.0	-	-	-
19	0.33239	18.349	12.250	6.099
18	0.05990	47.321	44.487	2.834
17	0.70687×10^{-2}	257.64	255.82	1.821
16	0.59968×10^{-2}	2227.82	2226.48	1.336
15	1.38591×10^{-4}	2.7291×10^{4}	2.7290×10^{4}	1.053
14	0.19484×10^{-5}	4.4575×10^{5}	4.4575×10^{5}	0.868
13	0.78908×10^{-7}	9.3607×10^{6}	9.3607×10^{6}	0.739
12	0.26010×10^{-8}	2.4698×10^{8}	2.4698×10^{8}	0.642
11	0.70434×10^{-10}	8.0680×10^{9}	8.0680×10^{9}	0.568

Table (5.[illegible]). Reliability results for the 20 identical machines example.

5.4 Load Model

The scope of this section is to incorporate a model of the power system load with the generation system model developed previously. Combination of this load and the generation model, permits computation of the availability, frequency of occurence and mean duration of margin states.

The load model is a Markov chain and it is defined by the following quantities:

Number of load levels	N
Description of load level, MW	L_i, $i = 1, 2, \ldots, N$
Number of occurrences of L_i	n_i, $i = 1, \ldots, N$
Interval length, days	$D = \sum_{i=1}^{N} n_i$
Expected duration or peak, days	$e < 1$
"Availability" of L_i, p.u.	$A_i = n_i e/D$
Transition rate to greater load	$\lambda_{+L_i} = 0$
Transition rate to lesser load	$\lambda_{-L_i} = 1/e$
Frequency of occurrence of L_i	$f_i = n_i/D$
For Low Load Period:	
Load state	L_o, MW
Availability	$A_o = 1 - e$
Transition rates	$\lambda_{-L_o} = 0$
	$\lambda_{+L_o} = \lambda_o = 1/(1 - e)$
Frequencv	$f_o = 1$

Fig.(5.5) shows a sequence of loads for a basic load model. The transfer of the system load from peak values to lower values will be developed as a Markov chain. Fig.(5.6) shows the state transition diagram for this basic load model.

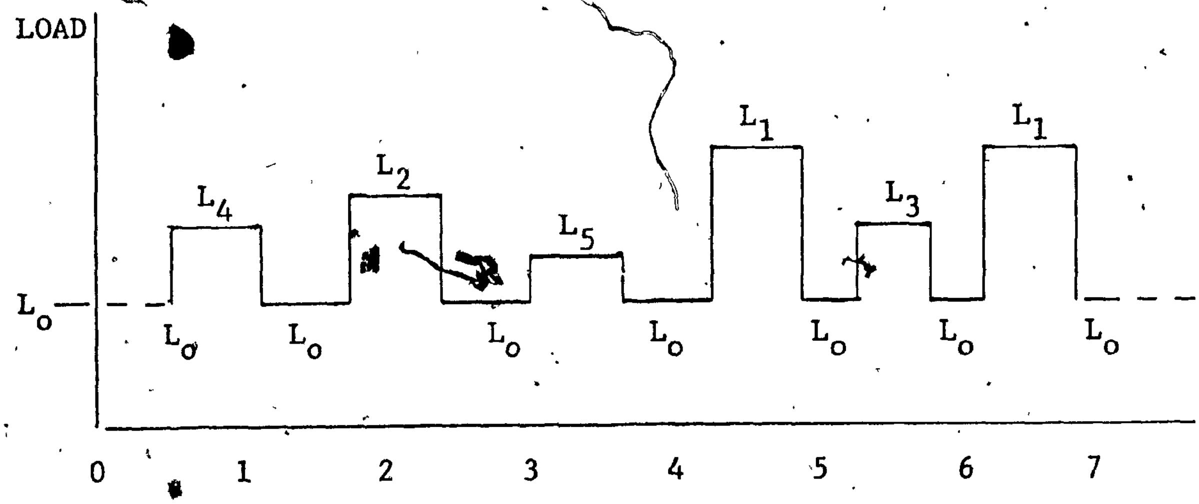

Fig. (5.5). Sequence of loads for a basic load model.

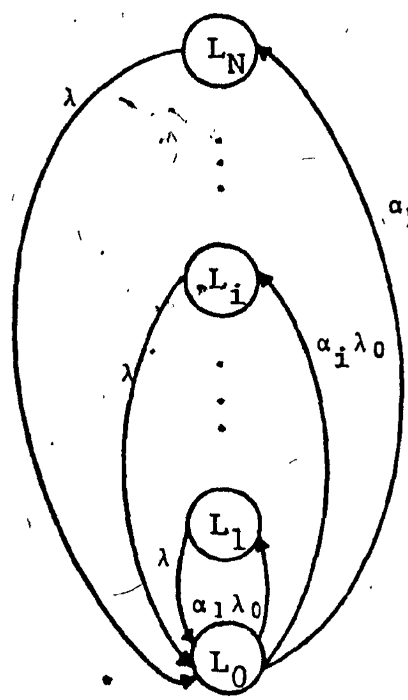

Fig. (5.6). State transition diagram for basic load model.

The Markov analysis of the chain gives:

$$\begin{bmatrix} \dot{P}_o \\ \dot{P}_1 \\ \vdots \\ \dot{P}_i \\ \vdots \\ \dot{P}_N \end{bmatrix} = \begin{bmatrix} -\lambda_0 & \lambda & \lambda & \cdots & & \lambda \\ \alpha_1\lambda_0 & -\lambda & 0 & & & 0 \\ \vdots & & & & & \\ \alpha_1\lambda_0 & 0 & 0 & \cdots & -\lambda & 0 \\ \vdots & & & & & \\ \alpha_N\lambda_0 & 0 & 0 & & & -\lambda \end{bmatrix} \begin{bmatrix} P_o \\ P_1 \\ \vdots \\ P_i \\ \vdots \\ P_N \end{bmatrix}$$

where:

$\alpha_1 = n_i/D, \quad i = 1, 2, \ldots, N,$

= the probability that the system presently at state L_o, will go to a peak L_i,

$\lambda = 1/e$ = transition rate downward from L_i,

$\alpha_0\lambda_0 = \alpha_1/(1 - e), \quad i = 1, 2, \ldots, N,$

= the transition state upward from L_o to load state L_i,

P_i = the probability that the system is in state L_i, for $i = 1, 2, \ldots, N$.

In the steady state, the vector $\dot{P}$ is zero, and,

$$\sum_{i=0}^{N} P_i = 1$$ must hold

The solution for this initial condition gives:

$$P_o = \frac{\lambda}{(\lambda + \lambda_0)} = 1 - e$$

and,

$$P_i = \frac{\alpha_i \lambda_0}{(\lambda + \lambda_0)} = \frac{n_i e}{D}$$

To illustrate the technique, assume that for a 20-day period, peak loads of 40, 25, 20 and 15 MW are expected to occur 2, 5, 8, and 5 days, respectively. Let the duration of each peak load e be 1/4 day and $\lambda = 4$ per day and $\lambda_0 = 4/3$ per day. The interval load model data are shown in table (5.8). Note that this particular set of data is for the 20-day interval. In order to obtain annual quantities, the steady state probability must be multiplied by 20/365.

State Number	Load L_i	Occurrences n_i (days)	Probabilities α_i (pu)	P_i	Transfer Rates λ_+ (per day)	λ_-
1	40	2	0.10	0.025	0	4
2	25	5	0.25	0.0625	0	4
3	20	8	0.40	0.10	0	4
4	15	5	0.25	0.0625	0	4
0	L_o	20	1	0.75	4/3	0

Table (5.8). Interval load model data.

5.5 Reserve Margin States

Reserve or margin is the difference between the available capacity and the load. Thus, a margin state m_k is the difference of the capacity state C_j and the load state L_i. i.e.

$$m_k = C_j - L_i \tag{5.14}$$

The rates of departure from m_k to larger or smaller margin states are given by the following two equations.

$$\lambda_{+m} = \lambda_{+c} + \lambda_{-L}$$

and,

$$\lambda_{-m} = \lambda_{-c} + \lambda_{+L}$$

where, the subscripts (+) and (-) indicate up and down transitions and the transcripts m, c and L designate margin, capacity and load states respectively.

The availability of the margin state is:

$$A_k = A_j A_i \quad \text{for the exact margin state}$$

$$A_k = \sum_{1=1}^{N} A_1 \quad \text{for the identical margin states}$$

and,

$$A_m = \sum_{L} A_L A_C \quad \text{for the cumulative margin states}$$

The departure rates are:

$$\lambda_{\pm k} = \lambda_{\pm j} + \lambda_{\pm i} \qquad \text{for the exact margin states}$$

$$\lambda_k = \sum_{1=1}^{N} A_1 \lambda_1 / A_k \qquad \text{for the identical margin states}$$

and finally, the frequencies of occurence are:

$$f_k = A_k (\lambda_{+k} + \lambda_{-k}) \text{ for the exact margin states}$$

$$f_k = \sum_{1=1}^{N} f_i \qquad \text{for the identical margin state}$$

and,

$$f_M = \sum_{L} A_L (f_G + A_G (\lambda_{-L} - \lambda_{+L}))$$

for the cumulative margin states.

The subscripts have the following meaning:

m = exact margin of M, MW

M = margin of M, MW or less

L = exact load of L, MW

C = exact capacity of C, MW

G = cumulative capacity of G, MW or less.

Having defined all of the above, a margin availability table can be constructed. The table includes such data as margin in MW, availability, and transfer rates. As an example, consider two generators, one of which has a capacity of 30 MW and the other 20 MW. The availability is 0.98 for both and the repair rates r = 2.040816. The load data are as mentioned in the previous section. The margin availability table for this example is shown in table (5.9).

Generation Data						Load Data			
					i	1	2	3	4
					L_i	40	25	20	15
					A_i	0.00273973	0.00684932	0.0109589	0.00684932
					λ_+	0	0	0	0
					λ_-	2	2	2	2
j	C_j	A_j	λ_+	λ_-					
1	50	0.9604	0	0.02		m=10	25	30	35
						A=0.002631	0.006578	0.010525	0.006578
						λ_+ = 2	2	2	2
						λ_- = 0.02	0.02	0.02	0.02
2	30	0.0196	0.49	0.01		m = -10	5	10	15
						A=0.000054	0.000134	0.000215	0.000134
						λ_+ = 2.49	2.49	2.49	2.49
						λ_- = 0.01	0.01	0.01	0.01
3	20	0.0196	0.49	0.01		m = -20	-5	0	5
						A=0.000054	0.000134	0.000215	0.000134
						λ_+ = 2.49	2.49	2.49	2.49
						λ_- = 0.01	0.01	0.01	0.01
4	0	0.0004	0.98	0		m = -40	-25	-20	-15
						A=0.000001	0.000003	0.000004	0.000003
						λ_+ = 2.98	2.98	2.98	2.98
						λ_- = 0	0	0	0

Table (5.9): Margin-availability table for the two-machine four-load -level example.

CHAPTER VI

THE RELIABILITY OF MECHANICAL AND CIVIL ENGINEERING SYSTEMS

6.1 Introduction

In the analysis of the reliability of mechanical and civil engineering systems, the most popular techniques among designers are the interference model of strength and stress distributions method,[23,24] the functions of random variables method[25] and the risk analysis method.[26,27]

The purpose of the first method is to obtain an estimate of reliability of mechanical components subject to fatigue failure. The method applied to components subjected to completely reversed cyclic bending axial or torsion loads and to components subjected to a combination of stress and cyclic load. The Weibull and the Normal distributions are the mathematical tools to achieve this.

The first section of this chapter gives the mathematics of the Weibull distribution along with an example illustrating the technique. The second section gives the mathematics of the normal distribution with a second example. The functions of random variables method is an extension of the first method.

The particular problem which can be solved by the method, is the adjustment of strength distribution parameters obtained under normal test conditions to account for such factors as load, size, surface condition and temperature. The method utilizes the concepts of the normal distribution function. Finally the risk analysis method applies to civil engineering systems subjected to various loads. Its purpose is to establish reliability based design criteria which would account for uncertainties in the variables of the system. The last section of this chapter presents the mathematical model along with an example of the risk analysis method.

6.2 The Weibull Distribution and its Application to the Interference Model of Strength and Stress Distributions Method

The Weibull distribution was developed by W.Weibull[6] and it is widely used in problems involving metal fatigue failures. The distribution is not derived from basic mathematical principles but it is rather an emperical approximation. It is described by three parameters as:

$$w(x) = \frac{\beta\,(x-\gamma)}{\alpha} \exp\left\{\frac{(x-\gamma)^{\beta}}{\alpha}\right\} \qquad (6.1)$$

where, $w(x)$ = Weibull density function

α = scale parameter

β = shape parameter

γ = location parameter

In general, the corresponding reliability function is given as:

$$R(x) = \exp\left(\frac{-(x-\gamma)^{\beta}}{\alpha} \right) \qquad \gamma < x < \infty \tag{6.2}$$

In the interference model of strength and stress distribution method however, a modified form of reliability is used, namely,

$$R(x) = \exp\left(-\left(\frac{x - x_o}{\theta - x_o} \right)^{\beta} \right) \qquad x_o < x < \infty \tag{6.3}$$

where, x is the variable

x_o is the lower bound of x

θ is the characteristic strength

β is the slope parameter

Comparing equations (6.2) and (6.3),

$$\gamma = x_o$$

$$\alpha = (\theta - x_o)^{\beta}$$

This transformation was done because most available data is given in terms of β, θ and x_o. To illustrate the application of the method, consider the following example.

An alluminum alloy 7079-T652 forged is subjected to reverse bending stresses of 162.6 MN/m^2 at ambient temperature. It is required to estimate the reliability of the component.

From tables (RADC-TR-68-403 p.233) and for these conditions,

$$\beta = 3.093$$

$$\theta = 28.96$$

$$x_o = 20.51$$

and substituting in equation (6.3), the reliability is,

$$R(x) = \exp\left(- \left(\frac{23.6 - 20.51}{28.96 - 20.51} \right)^{3.096} \right)$$

$$= \exp(-0.0537) = 0.9473$$

where, 23.6 is the bending stress of 162.6 MN/m^2 in ksi

6.3 The Normal Distribution

If X denotes the time to failure of a random variable of a device which fails according to the normal or Gaussian law, the probability density function of X is:

$$f(x)= \frac{1}{\sqrt{2\pi}\ \sigma} \exp\left(- \frac{(x - \xi)^2}{2\ \sigma^2} \right) \tag{6.4}$$

where, σ is the standard deviation

ξ is the mean deviation.

The failure rate of the normal distribution is increasing in X and hence, this distribution can be used to characterize wear of metals. One basic property of the normal distribution is the following. If X and Y are independent normal variables and Z is their product then:

$$\xi_z = \xi_x \cdot \xi_y \tag{6.5}$$

and

$$\sigma_z = \sqrt{\xi_x^2 \sigma_y^2 + \xi_y^2 \sigma_x^2 + \sigma_x^2 \sigma_y^2} \tag{6.6}$$

It will be shown later that the functions of random variables technique is based on this particular property.

In general the reliability function is given as:

$$R(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_t^\infty \exp\left(-\frac{(x-\xi)^2}{2\sigma^2}\right) dx \tag{6.7}$$

One indirect way to estimate the reliability will be shown in the next subsection.

6.3.1 The Normal Distribution and its Application to the Interference Model of Strength and Stress Distribution Method

An indirect way to calculate the reliability of two normally distributed parameters is achieved through the calculation of the unit normal deviate Z. Having calculated Z, the reliability is then obtained from tables. The unit normal deviate Z is defined as:

$$Z = \frac{x - \xi}{\sigma} \tag{6.8}$$

where, x is the random variable,

and, ξ, σ have been previously defined.

This method of calculating the reliability is called the interference model of strength and stress distribution method. As an example, let the aluminum alloy in the previous example be of type 5086-434 and the rest of the conditions be the same. From tables (RADC-TR-403 p.211) and for the given condition,

$$\xi = 22.39 \qquad \text{and} \qquad \sigma = 1.434$$

therefore from eq.(6.8),

$$Z = \frac{23.6 - 22.39}{1.43}$$

$$= 0.84$$

The corresponding reliability for this value of Z is obtained from table (RADC-TR-68-403 p. 7), as being equal to $R = 0.2$.

6.3.2 The Normal Distribution and its Application to the Function of Random Variables Method

The fatigue strength data is traditionally determined under standard test conditions, (completely reversed bending stress, room temperature, 0.3 inch specimen diameter, polished surface). The data are utilized for other conditons by adjusting the mean strength with several "standard" multiplicative factors. For example, if ξ_x is the observed mean strength at a given life.

$$\xi'_x = K_1 K_2 K_3 K_4 \xi_x \tag{6.9}$$

where,

K_1 = size factor (where component is of different diameter than test speciment),

K_2 = surface factor (to account for corrosion, notch effects, and other surface finishes),

K_3 = temperature factor,

K_4 = load factor (to account for axial, shear, or other load condition than bending).

To estimate fatigue strength parameters in service from standard data, let,

X = observed strength at a given life,

X' = strength in service at same life,

ξ_x = mean strength,

ξ'_x = mean strength in service,

σ_x = standard deviation of strength,

σ'_x = standard deviation in service,

K_i = multiplicative factor,

ξ_i = mean value of factor K_i,

σ_i = standard deviation of factor K_i.

Then, if,

$$X' = K_i X \tag{6.10}$$

it follows from equations (6.5) and (6.6), that,

$$\xi'_x = \xi_i \xi_x \tag{6.11}$$

$$\sigma'_x = \sqrt{\xi_x^2 \sigma_i^2 + \xi_i^2 \sigma_x^2 + \sigma_i^2 \sigma_x^2} \tag{6.12}$$

If X is the product of several K_i and X, the above formula may be applied repetitively, replacing X by X' and K_i by K_j, and so on.

Having calculated ξ'_x and σ'_x, the unit normal deviate Z can be found applying eq.(6.8), and consequently, the reliability can be obtained from the same tables as shown in the previous example in section 6.3.1.

6.4 The Risk Analysis Method

The purpose of this section is to illustrate the quantitative analysis of design uncertainties, and show how these uncertainties affect the level of risk. At the same time the risks associated with existing design procedures are evaluated with reference to reinforced concrete. These risks then serve as the initial basis for the formulation of risk-based design criteria.

The equation for calculating the failure risk or the probability of failure is:

$$p_f = P\left(R < S \right) = P\left(N_r \hat{R} < N_s \hat{S} \right) \quad (6.13)$$

where, $\hat{R}$ is the predicted theoretical model of resistance,

$\hat{S}$ is the predicted theoretical model of load,

N_r is the correction factor accounting for the imperfections in R,

N_s is the correction factor accounting for the imperfections in $\hat{S}$.

If the distribution of $\ln(R/S)$ is assumed to be normal, then p_f can take the form:

$$p_f = 1 - \Phi\left(\frac{\ln(\bar{R}/\bar{S})}{\sqrt{\Omega_R^2 + \Omega_S^2}} \right) \quad (6.14)$$

where,

$\bar{R}$ and $\bar{S}$ are the corresponding mean values for resistance and load respectively,

Ω_R and Ω_S are the total uncertaintities of resistance and load respectively, and

Φ indicates function, (normal distribution function).

As an example consider a reinforced concrete beam in flexure. The predicted mean capacity of this beam is then,

$$\overline{M}_T = \overline{A}_s\, \overline{f}_y\, \overline{d}\, \left(1 - \overline{\eta}\, \frac{\overline{A}_s}{\overline{b}\, \overline{d}}\, \frac{\overline{f}_y}{\overline{f}_c} \right) \tag{6.15}$$

where, $\overline{M}_T$ = the mean flexural capacity in tension mode,

$\overline{A}_s$ = the mean area of steel reinforcement,

$\overline{f}_y$ = the mean yield strength of reinforcing steel,

$\overline{d}$ = the mean depth to tension reinforcement

$\overline{\eta}$ = the mean concrete stress block parameter,

$\overline{b}$ = the mean width,

$\overline{f}_c$ = the mean concrete strength.

and the total uncertaintities in M_T are,

$$\Omega^2_{M_T} = \left(\frac{1 - 2\overline{\eta}\, q}{1 - \overline{\eta}\, q} \right) \left(\Omega^2_{f_y} + \Omega^2_{A_s} \right) + \\ + \left(\frac{\overline{\eta}\, q}{1 - \overline{\eta} q} \right) \left(\Omega^2_{f_c} + \Omega^2_{b} + \Omega^2_{\eta} \right) + \\ + \left(\frac{1}{1 - \overline{\eta}\, q} \right) \Omega^2_d \tag{6.16}$$

where,

$$q = \frac{\overline{A}_s}{\overline{b}\, \overline{d}}\, \frac{\overline{f}_y}{\overline{f}_c} = \overline{\rho}\, \frac{\overline{f}_y}{\overline{f}_c} \tag{6.17}$$

$\overline{\rho}$ = the mean design reinforcement ratio, and the subscripts of Ω indicate the associated parameter.

The uncertainties vary with concrete strength and yield strength of reinforcing steel. Their values are calculated with experiments. Table (6.1) shows the different values of Ω of a design with $f_y = 275.6$ MN/m^2 and $f_c = 20,67$ MN/m^2.

Parameter	Predicted mean	Total Uncertainty Ω
f_y(nominal 275.6 MN/m^2)	328.65	0.15
f_c(nominal 20.67 MN/m^2)	24.11	0.21
A_s		0.036
b		0.045
d		0.086
η	4.06	0.05

Table (6.1). Coefficients for Evaluation of Uncertainties in Flexural Capacity M_T.

Having found the total uncertainties, the probability of failure can be calculated according to equation (6.14). Fig.(6.1) shows the values of the probability as the mean design reinforcement ratio changes. The same method can be applied to calculate the probability of shear failure. Thus having defined both failure risks and employing a safety factor according to standards a complete design of a steel reinforced concrete can be achieved.

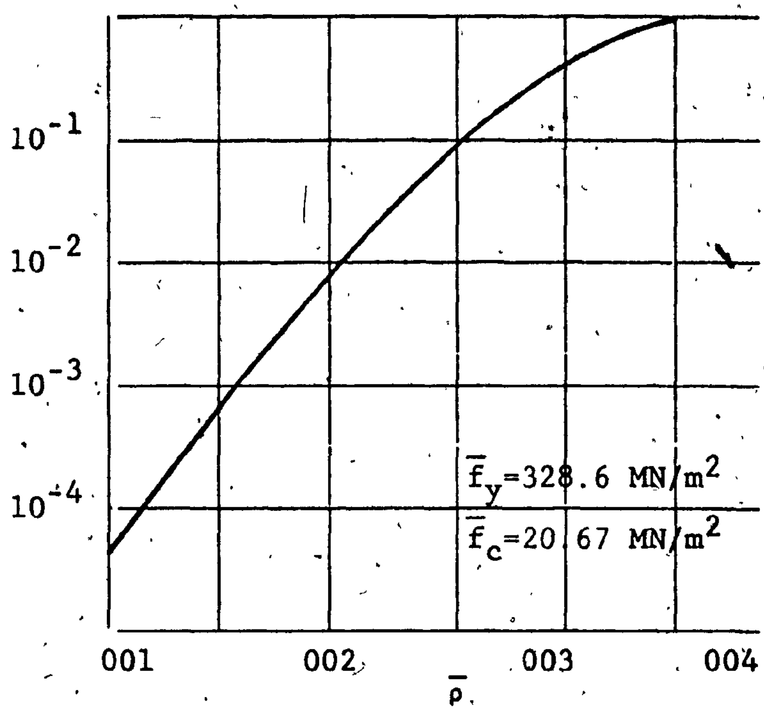

Fig.(6.1). Probability that failure will be compresive.

CHAPTER VII

CONCLUSION

The reliability of engineering systems was studied, definitions of the reliability indexes were given and the various techniques to compute the reliability of engineering systems were reviewed.

In the case of electronic components reliability, the part failure rate method was found to be very accurate and readily applicable to most electronic circuits.

In the reliability of power distribution systems a lack of variety of relevant papers was noticed. This is probably due to the fact that many writers tend to incorporate some of the elements of power distribution systems with either the power transmission system of the power generation system. The most applicable method for obtaining the reliability of a power distribution system is a variation of the part failure method. Two methods find extensive use in the reliability of transmission lines, the Markov processes method and the approximate method. Both methods were presented and covered even though it was hard to compare both methods for accuracy. It was very obvious that the approximate method is much simpler as far as calculations are concerned.

Three methods are available in estimating the reliability of power generation systems. The recursive technique method was selected because of its unique ability to consider load requirements and availability of power at the same time.

Finally in the reliability analysis of mechanical and civil engineering systems, it was found that the Weibull and the Normal distributions are the most commonly used distributions to develop techniques leading to reliability formulas.

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